CHAPTER 2
Polynomial and Rational Functions

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Section 2.1 Quadratic Functions and Models

You should know the following facts about parabolas.

- $f(x) = ax^2 + bx + c,\ a \neq 0$, is a quadratic function, and its graph is a parabola.
- If $a > 0$, the parabola opens upward and the vertex is the point with the minimum y-value.
  If $a < 0$, the parabola opens downward and the vertex is the point with the maximum y-value.
- The vertex is $(-b/2a, f(-b/2a))$.
- To find the x-intercepts (if any), solve
  $ax^2 + bx + c = 0.$
- The standard form of the equation of a parabola is
  $f(x) = a(x - h)^2 + k$
  where $a \neq 0$.
  (a) The vertex is (h, k).
  (b) The axis is the vertical line $x = h$.

Vocabulary Check

1. nonnegative integer; real

2. quadratic; parabola

3. axis or axis of symmetry

4. positive; minimum

5. negative; maximum

1. $f(x) = (x - 2)^2$ opens upward and has vertex $(2, 0)$. Matches graph (g).

2. $f(x) = (x + 4)^2$ opens upward and has vertex $(-4, 0)$. Matches graph (c).

3. $f(x) = x^2 - 2$ opens upward and has vertex $(0, -2)$. Matches graph (b).

4. $f(x) = 3 - x^2$ opens downward and has vertex $(0, 3)$. Matches graph (h).

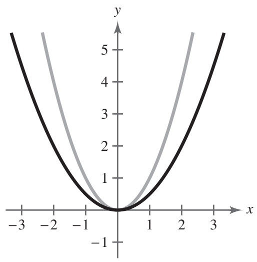
5. $f(x) = 4 - (x - 2)^2 = -(x - 2)^2 + 4$ opens downward and has vertex $(2, 4)$. Matches graph (f).

6. $f(x) = (x + 1)^2 - 2$ opens upward and has vertex $(-1, -2)$. Matches graph (a).

7. $f(x) = -(x - 3)^2 - 2$ opens downward and has vertex $(3, -2)$. Matches graph (e).

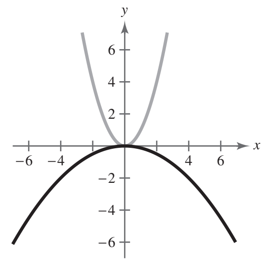
8. $f(x) = -(x - 4)^2$ opens downward and has vertex $(4, 0)$. Matches graph (d).

9. (a) $y = \frac{1}{2}x^2$



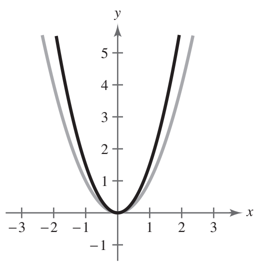
Vertical shrink

(b) $y = -\frac{1}{8}x^2$



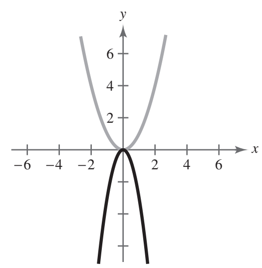
Vertical shrink and reflection in the x-axis

(c) $y = \frac{3}{2}x^2$



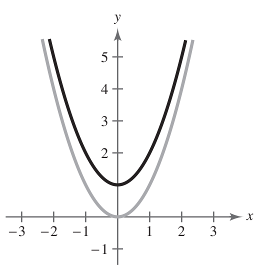
Vertical stretch

(d) $y = -3x^2$



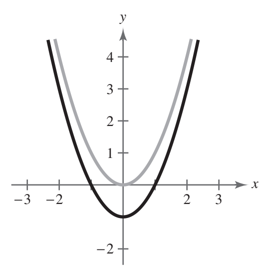
Vertical stretch and reflection in the x-axis

10. (a) $y = x^2 + 1$



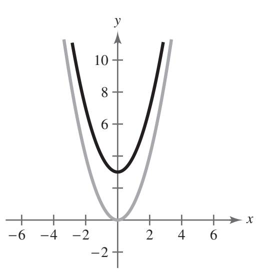
Vertical translation one unit upward

(b) $y = x^2 - 1$



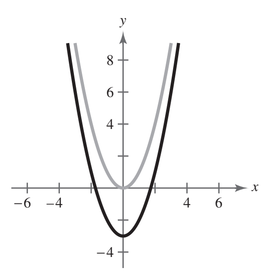
Vertical translation one unit downward

(c) $y = x^2 + 3$



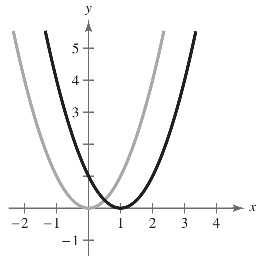
Vertical translation three units upward

(d) $y = x^2 - 3$



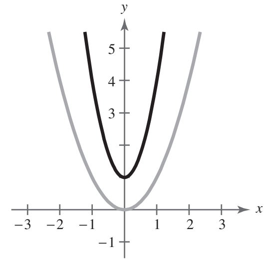
Vertical translation three units downward

11. (a) $y = (x - 1)^2$



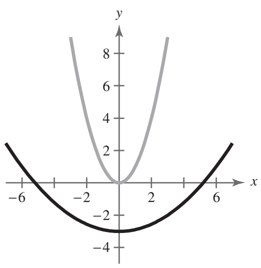
Horizontal translation one unit to the right

(b) $y = (3x)^2 + 1$



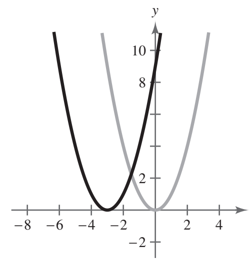
Horizontal shrink and a vertical translation one unit upward

(c) $y = \left(\frac{1}{3}x\right)^2 - 3$



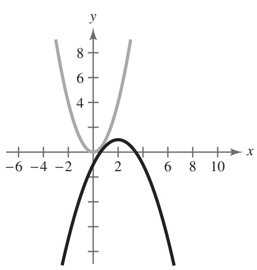
Horizontal stretch and a vertical translation three units downward

(d) $y = (x + 3)^2$



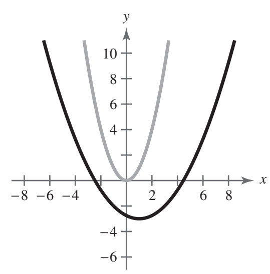
Horizontal translation three units to the left

12. (a) $y = -\frac{1}{2}(x - 2)^2 + 1$



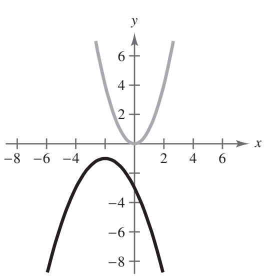
Horizontal translation two units to the right, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), reflection in the x-axis, and vertical translation one unit upward

(b) $y = \left[\frac{1}{2}(x - 1)\right]^2 - 3$



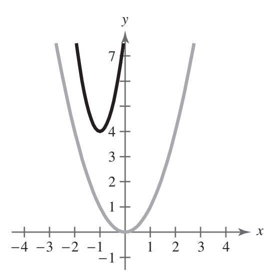
Horizontal translation one unit to the right, horizontal stretch (each x-value is multiplied by 2), and vertical translation three units downward

(c) $y = -\frac{1}{2}(x + 2)^2 - 1$



Horizontal translation two units to the left, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), reflection in x-axis, and vertical translation one unit downward

(d) $y = [2(x + 1)]^2 + 4$



Horizontal translation one unit to the left, horizontal shrink (each x-value is multiplied by $\frac{1}{2}$), and vertical translation four units upward

13. $f(x) = x^2 - 5$

Vertex: $(0, -5)$

Axis of symmetry: $x = 0$ or the y-axis

Find x-intercepts:

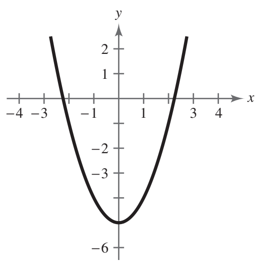
$x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

x-intercepts:

$\left(-\sqrt{5}, 0\right), \left(\sqrt{5}, 0\right)$



14. $h(x) = 25 - x^2$

Vertex: $(0, 25)$

Axis of symmetry: $x = 0$

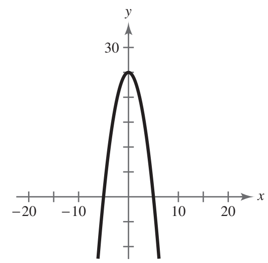
Find x-intercepts:

$25 - x^2 = 0$

$x^2 = 25$

$x = \pm 5$

x-intercepts: $(\pm 5, 0)$



15. $f(x) = \frac{1}{2}x^2 - 4 = \frac{1}{2}(x - 0)^2 - 4$

Vertex: $(0, -4)$

Axis of symmetry: $x = 0$ or the y-axis

Find x-intercepts:

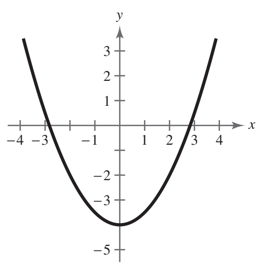
$\frac{1}{2}x^2 - 4 = 0$

$x^2 = 8$

$x = \pm\sqrt{8} = \pm 2\sqrt{2}$

x-intercepts:

$\left(-2\sqrt{2}, 0\right), \left(2\sqrt{2}, 0\right)$



16. $f(x) = 16 = \frac{1}{4}x^2 = -\frac{1}{4}x^2 + 16$

Vertex: $(0, 16)$

Axis of symmetry: $x = 0$

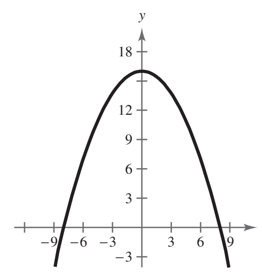
Find x-intercepts:

$16 - \frac{1}{4}x^2 = 0$

$x^2 = 64$

$x = \pm 8$

x-intercepts: $(\pm 8, 0)$



17. $f(x) = (x + 5)^2 - 6$

Vertex: $(-5, -6)$

Axis of symmetry: $x = -5$

Find x-intercepts:

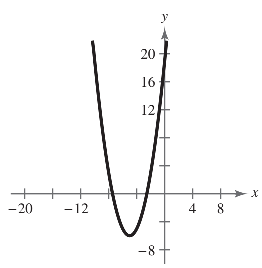
$(x + 5)^2 - 6 = 0$

$(x + 5)^2 = 6$

$x + 5 = \pm\sqrt{6}$

$x = -5 \pm \sqrt{6}$

x-intercepts: $\left(-5 - \sqrt{6}, 0\right), \left(-5 + \sqrt{6}, 0\right)$



18. $f(x) = (x - 6)^2 + 3$

Vertex: $(6, 3)$

Axis of symmetry: $x = 6$

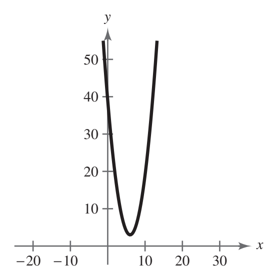
Find x-intercepts:

$(x - 6)^2 + 3 = 0$

$(x - 6)^2 = -3$

Not possible for real x

No x-intercepts

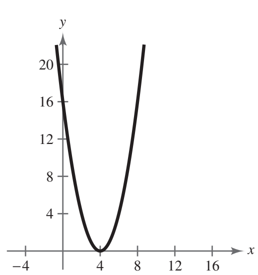


19. $h(x) = x^2 - 8x + 16 = (x - 4)^2$

Vertex: $(4, 0)$

Axis of symmetry: $x = 4$

x-intercept: $(4, 0)$

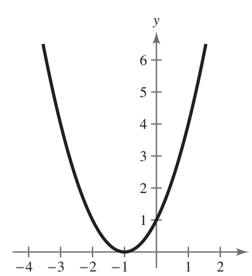


20. $g(x) = x^2 + 2x + 1 = (x + 1)^2$

Vertex: $(-1, 0)$

Axis of symmetry: $x = -1$

x-intercept: $(-1, 0)$



21. $f(x) = x^2 - x + \frac{5}{4}$

$$= \left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} + \frac{5}{4}$$

$$= \left(x - \frac{1}{2}\right)^2 + 1$$

Vertex: $\left(\frac{1}{2}, 1\right)$

Axis of symmetry: $x = \frac{1}{2}$

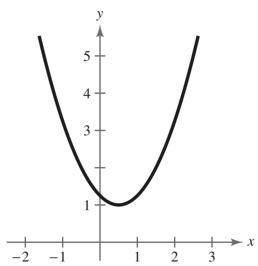
Find x-intercepts:

$x^2 - x + \frac{5}{4} = 0$

$$x = \frac{1 \pm \sqrt{1 - 5}}{2}$$

Not a real number

No x-intercepts



22. $f(x) = x^2 + 3x + \frac{1}{4}$

$$= \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + \frac{1}{4}$$

$$= \left(x + \frac{3}{2}\right)^2 - 2$$

Vertex: $\left(-\frac{3}{2}, -2\right)$

Axis of symmetry: $x = -\frac{3}{2}$

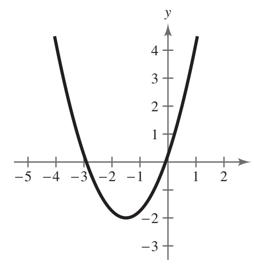
Find x-intercepts:

$x^2 + 3x + \frac{1}{4} = 0$

$$x = \frac{-3 \pm \sqrt{9 - 1}}{2}$$

$$= -\frac{3}{2} \pm \sqrt{2}$$

x-intercepts: $\left(-\frac{3}{2} \pm \sqrt{2}, 0\right)$



23. $f(x) = -x^2 + 2x + 5$

$$= -(x^2 - 2x + 1) - (-1) + 5$$

$$= -(x - 1)^2 + 6$$

Vertex: $(1, 6)$

Axis of symmetry: $x = 1$

Find x-intercepts:

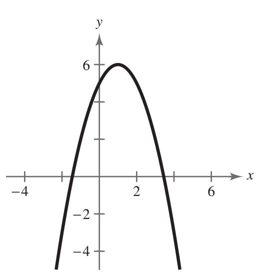
$-x^2 + 2x + 5 = 0$

$x^2 - 2x - 5 = 0$

$$x = \frac{2 \pm \sqrt{4 + 20}}{2}$$

$$= 1 \pm \sqrt{6}$$

x-intercepts: $\left(1 - \sqrt{6}, 0\right), \left(1 + \sqrt{6}, 0\right)$



24. $f(x) = -x^2 - 4x + 1 = -(x^2 + 4x) + 1$

$$= -(x^2 + 4x + 4) - (-4) + 1$$

$$= -(x + 2)^2 + 5$$

Vertex: $(-2, 5)$

Axis of symmetry: $x = -2$

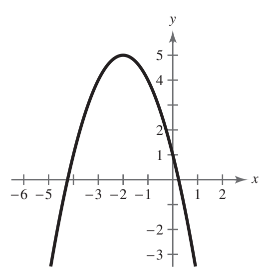
Find x-intercepts: $-x^2 - 4x + 1 = 0$

$x^2 + 4x - 1 = 0$

$$x = \frac{-4 \pm \sqrt{16 + 4}}{2}$$

$$= -2 \pm \sqrt{5}$$

x-intercepts: $\left(-2 \pm \sqrt{5}, 0\right)$



25. $h(x) = 4x^2 - 4x + 21$

$= 4\left(x^2 - x + \frac{1}{4}\right) - 4\left(\frac{1}{4}\right) + 21$

$= 4\left(x - \frac{1}{2}\right)^2 + 20$

Vertex: $\left(\frac{1}{2}, 20\right)$

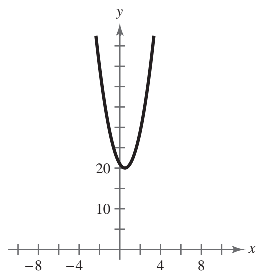
Axis of symmetry: $x = \frac{1}{2}$

Find x-intercepts:

$4x^2 - 4x + 21 = 0$

$x = \dfrac{4 \pm \sqrt{16 - 336}}{2(4)}$

Not a real number $\Longrightarrow$ No x-intercepts



26. $f(x) = 2x^2 - x + 1$

$= 2\left(x^2 - \frac{1}{2}x\right) + 1$

$= 2\left(x - \frac{1}{4}\right)^2 - 2\left(\frac{1}{16}\right) + 1$

$= 2\left(x - \frac{1}{4}\right)^2 + \frac{7}{8}$

Vertex: $\left(\frac{1}{4}, \frac{7}{8}\right)$

Axis of symmetry: $x = \frac{1}{4}$

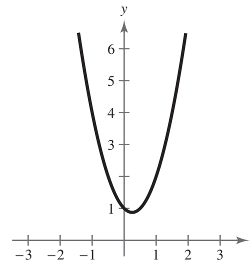
Find x-intercepts:

$2x^2 - x + 1 = 0$

$x = \dfrac{1 \pm \sqrt{1 - 8}}{2(2)}$

Not a real number

No x-intercepts



27. $f(x) = \frac{1}{4}x^2 - 2x - 12$

$= \frac{1}{4}(x^2 - 8x + 16) - \frac{1}{4}(16) - 12$

$= \frac{1}{4}(x - 4)^2 - 16$

Vertex: $(4, -16)$

Axis of symmetry: $x = 4$

Find x-intercepts:

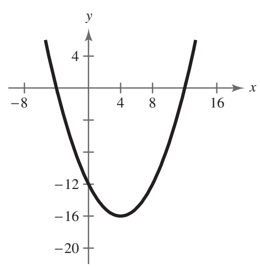
$\frac{1}{4}x^2 - 2x - 12 = 0$

$x^2 - 8x - 48 = 0$

$(x + 4)(x - 12) = 0$

$x = -4$ or $x = 12$

x-intercepts: $(-4, 0), (12, 0)$



28. $f(x) = -\frac{1}{3}x^2 + 3x - 6$

$= -\frac{1}{3}(x^2 - 9x) - 6$

$= -\frac{1}{3}\left(x^2 - 9x + \frac{81}{4}\right) + \frac{1}{3}\left(\frac{81}{4}\right) - 6$

$= -\frac{1}{3}\left(x - \frac{9}{2}\right)^2 + \frac{3}{4}$

Vertex: $\left(\frac{9}{2}, \frac{3}{4}\right)$

Axis of symmetry: $x = \frac{9}{2}$

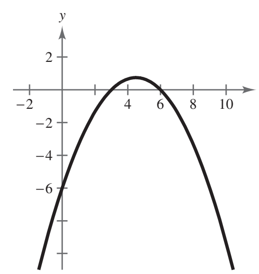
Find x-intercepts:

$-\frac{1}{3}x^2 + 3x - 6 = 0$

$x^2 - 9x + 18 = 0$

$(x - 3)(x - 6) = 0$

x-intercepts: $(3, 0), (6, 0)$

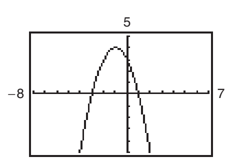


29. $f(x) = -(x^2 + 2x - 3) = -(x + 1)^2 + 4$

Vertex: $(-1, 4)$

Axis of symmetry: $x = -1$

x-intercepts: $(-3, 0), (1, 0)$



30. $f(x) = -(x^2 + x - 30)$

$= -(x^2 + x) + 30$

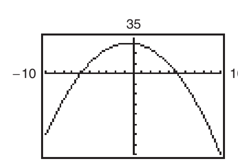
$= -\left(x^2 + x + \frac{1}{4}\right) + \frac{1}{4} + 30$

$= -\left(x + \frac{1}{2}\right)^2 + \frac{121}{4}$

Vertex: $\left(-\frac{1}{2}, \frac{121}{4}\right)$

Axis of symmetry: $x = -\frac{1}{2}$

x-intercepts: $(-6, 0), (5, 0)$

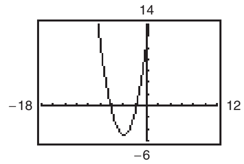


31. $g(x) = x^2 + 8x + 11 = (x + 4)^2 - 5$

Vertex: $(-4, -5)$

Axis of symmetry: $x = -4$

x-intercepts: $\left(-4 \pm \sqrt{5}, 0\right)$



32. $f(x) = x^2 + 10x + 14$

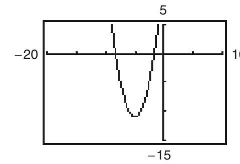
$= (x^2 + 10x + 25) - 25 + 14$

$= (x + 5)^2 - 11$

Vertex: $(-5, -11)$

Axis of symmetry: $x = -5$

x-intercepts: $\left(-5 \pm \sqrt{11}, 0\right)$



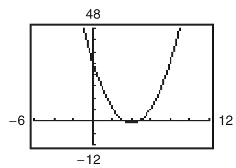
33. $f(x) = 2x^2 - 16x + 31$

$= 2(x - 4)^2 - 1$

Vertex: $(4, -1)$

Axis of symmetry: $x = 4$

x-intercepts: $\left(4 \pm \frac{1}{2}\sqrt{2}, 0\right)$



34. $f(x) = -4x^2 + 24x - 41$

$= -4(x^2 - 6x) - 41$

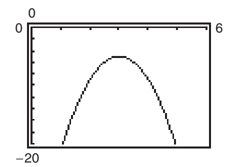
$= -4(x^2 - 6x + 9) + 36 - 41$

$= -4(x - 3)^2 - 5$

Vertex: $(3, -5)$

Axis of symmetry: $x = 3$

No x-intercepts

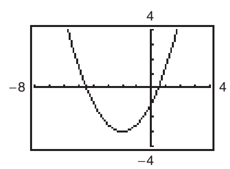


35. $g(x) = \frac{1}{2}(x^2 + 4x - 2) = \frac{1}{2}(x + 2)^2 - 3$

Vertex: $(-2, -3)$

Axis of symmetry: $x = -2$

x-intercepts: $\left(-2 \pm \sqrt{6}, 0\right)$



36. $f(x) = \frac{3}{5}(x^2 + 6x - 5)$

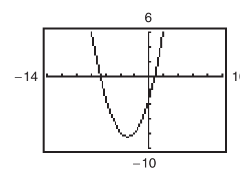
$= \frac{3}{5}(x^2 + 6x + 9) - \frac{27}{5} - 3$

$= \frac{3}{5}(x + 3)^2 - \frac{42}{5}$

Vertex: $\left(-3, -\frac{42}{5}\right)$

Axis of symmetry: $x = -3$

x-intercepts: $\left(-3 \pm \sqrt{14}, 0\right)$



37. $(1, 0)$ is the vertex.

$y = a(x - 1)^2 + 0 = a(x - 1)^2$

Since the graph passes through the point $(0, 1)$, we have:

$1 = a(0 - 1)^2$

$1 = a$

$y = 1(x - 1)^2 = (x - 1)^2$

38. $(0, 1)$ is the vertex.

$f(x) = a(x - 0)^2 + 1 = ax^2 + 1$

Since the graph passes through $(1, 0)$,

$0 = a(1)^2 + 1$

$-1 = a.$

So, $y = -x^2 + 1$.

39. $(-1, 4)$ is the vertex.

$y = a(x + 1)^2 + 4$

Since the graph passes through the point $(1, 0)$, we have:

$0 = a(1 + 1)^2 + 4$

$-4 = 4a$

$-1 = a$

$y = -1(x + 1)^2 + 4 = -(x + 1)^2 + 4$

40. $(-2, -1)$ is the vertex.

$f(x) = a(x + 2)^2 - 1$

Since the graph passes through $(0, 3)$,

$3 = a(0 + 2)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a.$

So, $y = (x + 2)^2 - 1$.

41. $(-2, 2)$ is the vertex.

$y = a(x + 2)^2 + 2$

Since the graph passes through the point $(-1, 0)$, we have:

$0 = a(-1 + 2)^2 + 2$

$-2 = a$

$y = -2(x + 2)^2 + 2$

42. $(2, 0)$ is the vertex.

$f(x) = a(x - 2)^2 + 0 = a(x - 2)^2$

Since the graph passes through $(3, 2)$,

$2 = a(3 - 2)^2$

$2 = a.$

So, $y = 2(x - 2)^2$.

43. $(-2, 5)$ is the vertex.

$f(x) = a(x + 2)^2 + 5$

Since the graph passes through the point $(0, 9)$, we have:

$9 = a(0 + 2)^2 + 5$

$4 = 4a$

$1 = a$

$f(x) = 1(x + 2)^2 + 5 = (x + 2)^2 + 5$

44. $(4, -1)$ is the vertex.

$f(x) = a(x - 4)^2 - 1$

Since the graph passes through $(2, 3)$,

$3 = a(2 - 4)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a.$

So, $f(x) = (x - 4)^2 - 1$.

45. $(3, 4)$ is the vertex.

$f(x) = a(x - 3)^2 + 4$

Since the graph passes through the point $(1, 2)$, we have:

$2 = a(1 - 3)^2 + 4$

$-2 = 4a$

$-\frac{1}{2} = a$

$f(x) = -\frac{1}{2}(x - 3)^2 + 4$

46. $(2, 3)$ is the vertex.

$f(x) = a(x - 2)^2 + 3$

Since the graph passes through $(0, 2)$,

$2 = a(0 - 2)^2 + 3$

$2 = 4a + 3$

$-1 = 4a$

$-\frac{1}{4} = a.$

So, $f(x) = -\frac{1}{4}(x - 2)^2 + 3$.

47. $(5, 12)$ is the vertex.

$f(x) = a(x - 5)^2 + 12$

Since the graph passes through the point $(7, 15)$, we have:

$15 = a(7 - 5)^2 + 12$

$3 = 4a \implies a = \frac{3}{4}$

$f(x) = \frac{3}{4}(x - 5)^2 + 12$

48. $(-2, -2)$ is the vertex.

$f(x) = a(x + 2)^2 - 2$

Since the graph passes through $(-1, 0)$,

$0 = a(-1 + 2)^2 - 2$

$0 = a - 2$

$2 = a.$

So, $f(x) = 2(x + 2)^2 - 2$.

49. $\left(-\frac{1}{4}, \frac{3}{2}\right)$ is the vertex.

$f(x) = a\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}$

Since the graph passes through the point $(-2, 0)$, we have:

$0 = a\left(-2 + \frac{1}{4}\right)^2 + \frac{3}{2}$

$-\frac{3}{2} = \frac{49}{16}a \implies a = -\frac{24}{49}$

$f(x) = -\frac{24}{49}\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}$

50. $\left(\frac{5}{2}, -\frac{3}{4}\right)$ is the vertex.

$f(x) = a\left(x - \frac{5}{2}\right)^2 - \frac{3}{4}$

Since the graph passes through $(-2, 4)$,

$4 = a\left(-2 - \frac{5}{2}\right)^2 - \frac{3}{4}$

$4 = \frac{81}{4}a - \frac{3}{4}$

$\frac{19}{4} = \frac{81}{4}a$

$\frac{19}{81} = a.$

So, $f(x) = \frac{19}{81}\left(x - \frac{5}{2}\right)^2 - \frac{3}{4}$.

51. $\left(-\frac{5}{2}, 0\right)$ is the vertex.

$f(x) = a\left(x + \frac{5}{2}\right)^2$

Since the graph passes through the point $\left(-\frac{7}{2}, -\frac{16}{3}\right)$, we have:

$-\frac{16}{3} = a\left(-\frac{7}{2} + \frac{5}{2}\right)^2$

$-\frac{16}{3} = a$

$f(x) = -\frac{16}{3}\left(x + \frac{5}{2}\right)^2$

52. $(6, 6)$ is the vertex.

$f(x) = a(x - 6)^2 + 6$

Since the graph passes through $\left(\frac{61}{10}, \frac{3}{2}\right)$,

$\frac{3}{2} = a\left(\frac{61}{10} - 6\right)^2 + 6$

$\frac{3}{2} = \frac{1}{100}a + 6$

$-\frac{9}{2} = \frac{1}{100}a$

$-450 = a.$

So, $f(x) = -450(x - 6)^2 + 6.$

53. $y = x^2 - 16$

x-intercepts: $(\pm 4, 0)$

$0 = x^2 - 16$

$x^2 = 16$

$x = \pm 4$

54. $y = x^2 - 6x + 9$

x-intercept: $(3, 0)$

$0 = x^2 - 6x + 9$

$0 = (x - 3)^2$

$x - 3 = 0 \implies x = 3$

55. $y = x^2 - 4x - 5$

x-intercepts: $(5, 0), (-1, 0)$

$0 = x^2 - 4x - 5$

$0 = (x - 5)(x + 1)$

$x = 5$ or $x = -1$

56. $y = 2x^2 + 5x - 3$

x-intercepts: $\left(\frac{1}{2}, 0\right), (-3, 0)$

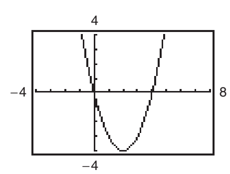
$0 = 2x^2 + 5x - 3$

$0 = (2x - 1)(x + 3)$

$2x - 1 = 0 \implies x = \frac{1}{2}$

$x + 3 = 0 \implies x = -3$

57. $f(x) = x^2 - 4x$



x-intercepts: $(0, 0), (4,0)$

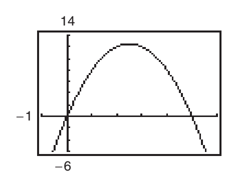
$0 = x^2 - 4x$

$0 = x(x - 4)$

$x = 0$ or $x = 4$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

58. $f(x) = -2x^2 + 10x$



x-intercepts: $(0, 0), (5, 0)$

$0 = -2x^2 + 10x$

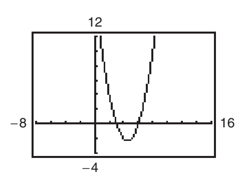
$0 = -2x(x - 5)$

$-2x = 0 \implies x = 0$

$x - 5 = 0 \implies x = 5$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

59. $f(x) = x^2 - 9x + 18$



x-intercepts: $(3, 0), (6, 0)$

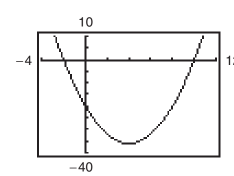
$0 = x^2 - 9x + 18$

$0 = (x - 3)(x - 6)$

$x = 3$ or $x = 6$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

60. $f(x) = x^2 - 8x - 20$



x-intercepts: $(-2, 0), (10, 0)$

$0 = x^2 - 8x - 20$

$0 = (x + 2)(x - 10)$

$x + 2 = 0 \implies x = -2$

$x - 10 = 0 \implies x = 10$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

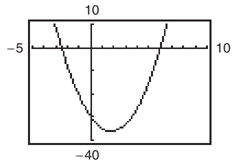
61. $f(x) = 2x^2 - 7x - 30$

x-intercepts: $\left(-\frac{5}{2}, 0\right), (6, 0)$

$0 = 2x^2 - 7x - 30$

$0 = (2x + 5)(x - 6)$

$x = -\frac{5}{2}$ or $x = 6$



The x-intercepts and the solutions of $f(x) = 0$ are the same.

62. $f(x) = 4x^2 + 25x - 21$

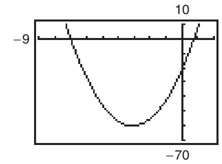
x-intercepts: $(-7, 0), \left(\frac{3}{4}, 0\right)$

$0 = 4x^2 + 25x - 21$

$0 = (x + 7)(4x - 3)$

$x + 7 = 0 \implies x = -7$

$4x - 3 = 0 \implies x = \frac{3}{4}$



The x-intercepts and the solutions of $f(x) = 0$ are the same.

63. $f(x) = -\frac{1}{2}(x^2 - 6x - 7)$

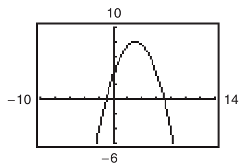
x-intercepts: $(-1, 0), (7, 0)$

$0 = -\frac{1}{2}(x^2 - 6x - 7)$

$0 = x^2 - 6x - 7$

$0 = (x + 1)(x - 7)$

$x = -1$ or $x = 7$



The x-intercepts and the solutions of $f(x) = 0$ are the same.

64. $f(x) = \frac{7}{10}(x^2 + 12x - 45)$

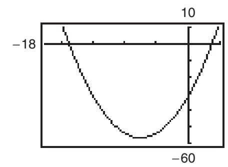
x-intercepts: $(-15, 0), (3, 0)$

$0 = \frac{7}{10}(x^2 + 12x - 45)$

$0 = (x + 15)(x - 3)$

$x + 15 = 0 \implies x = -15$

$x - 3 = 0 \implies x = 3$



The x-intercepts and the solutions of $f(x) = 0$ are the same.

65. $f(x) = [x - (-1)](x - 3)$ opens upward

$= (x + 1)(x - 3)$

$= x^2 - 2x - 3$

$g(x) = -[x - (-1)](x - 3)$ opens downward

$= -(x + 1)(x - 3)$

$= -(x^2 - 2x - 3)$

$= -x^2 + 2x + 3$

Note: $f(x) = a(x + 1)(x - 3)$ has x-intercepts $(-1, 0)$ and $(3, 0)$ for all real numbers $a \neq 0$.

66. $f(x) = [x - (-5)](x - 5)$

$= (x + 5)(x - 5)$

$= x^2 - 25$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -x^2 + 25$

Note: $f(x) = a(x^2 - 25)$ has x-intercepts $(-5, 0)$ and $(5, 0)$ for all real numbers $a \neq 0$.

67. $f(x) = (x - 0)(x - 10)$ opens upward

$= x^2 - 10x$

$g(x) = -(x - 0)(x - 10)$ opens downward

$= -x^2 + 10x$

Note: $f(x) = a(x - 0)(x - 10) = ax(x - 10)$ has x-intercepts $(0, 0)$ and $(10, 0)$ for all real numbers $a \neq 0$.

68. $f(x) = (x - 4)(x - 8)$

$= x^2 - 12x + 32$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -x^2 + 12x - 32$

Note: $f(x) = a(x - 4)(x - 8)$ has x-intercepts $(4, 0)$ and $(8, 0)$ for all real numbers $a \neq 0$.

69. $f(x) = [x - (-3)]\left[x - \left(-\frac{1}{2}\right)\right](2)$ opens upward

$= (x + 3)\left(x + \frac{1}{2}\right)(2)$

$= (x + 3)(2x + 1)$

$= 2x^2 + 7x + 3$

$g(x) = -(2x^2 + 7x + 3)$ opens downward

$= -2x^2 - 7x - 3$

Note: $f(x) = a(x + 3)(2x + 1)$ has x-intercepts $(-3, 0)$ and $\left(-\frac{1}{2}, 0\right)$ for all real numbers $a \neq 0$.

70. $f(x) = 2\left[x - \left(-\frac{5}{2}\right)\right](x - 2)$

$= 2\left(x + \frac{5}{2}\right)(x - 2)$

$= 2\left(x^2 + \frac{1}{2}x - 5\right)$

$= 2x^2 + x - 10$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -2x^2 - x + 10$

Note: $f(x) = a\left(x + \frac{5}{2}\right)(x - 2)$ has x-intercepts $\left(-\frac{5}{2}, 0\right)$ and $(2, 0)$ for all real numbers $a \neq 0$.

71. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + y = 110 \implies y = 110 - x.$

The product is $P(x) = xy = x(110 - x) = 110x - x^2$.

$$\begin{aligned} P(x) &= -x^2 + 110x \\ &= -(x^2 - 110x + 3025 - 3025) \\ &= -[(x - 55)^2 - 3025] \\ &= -(x - 55)^2 + 3025 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55$.

72. Let $x =$ first number and $y =$ second number. Then, $x + y = S$, $y = S - x$. The product is

$P(x) = xy = x(S - x).$

$$\begin{aligned} P(x) &= Sx - x^2 \\ &= -x^2 + Sx \\ &= -\left(x^2 - Sx + \frac{S^2}{4} - \frac{S^2}{4}\right) \\ &= -\left(x - \frac{S}{2}\right)^2 + \frac{S^2}{4} \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is $S^2/4$. This happens when $x = y = S/2$.

73. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + 2y = 24 \implies y = \dfrac{24 - x}{2}.$

The product is $P(x) = xy = x\left(\dfrac{24 - x}{2}\right)$.

$$\begin{aligned} P(x) &= \frac{1}{2}(-x^2 + 24x) \\ &= -\frac{1}{2}(x^2 - 24x + 144 - 144) \\ &= -\frac{1}{2}[(x - 12)^2 - 144] = -\frac{1}{2}(x - 12)^2 + 72 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 72. This happens when $x = 12$ and $y = (24 - 12)/2 = 6$. Thus, the numbers are 12 and 6.

74. Let $x =$ the first number and $y =$ the second number.

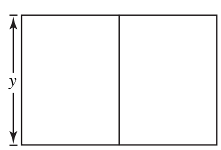
Then the sum is $x + 3y = 42 \implies y = \dfrac{42 - x}{3}$.

The product is $P(x) = xy = x\left(\dfrac{42 - x}{3}\right)$.

$$\begin{aligned} P(x) &= \frac{1}{3}(-x^2 + 42x) \\ &= -\frac{1}{3}(x^2 - 42x + 441 - 441) \\ &= -\frac{1}{3}[(x - 21)^2 - 441] = -\frac{1}{3}(x - 21)^2 + 147 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 147. This happens when $x = 21$ and $y = \dfrac{42 - 21}{3} = 7$. Thus, the numbers are 21 and 7.

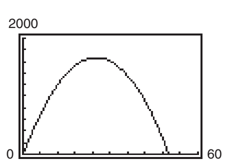
75. (a)



$4x + 3y = 200 \implies y = \dfrac{1}{3}(200 - 4x) = \dfrac{4}{3}(50 - x)$

$A = 2xy = 2x\left[\dfrac{4}{3}(50 - x)\right] = \dfrac{8}{3}x(50 - x) = \dfrac{8x(50 - x)}{3}$

(c)



This area is maximum when $x = 25$ feet and $y = \frac{100}{3} = 33\frac{1}{3}$ feet.

(b)

x	A
5	600
10	$1066\frac{2}{3}$
15	1400
20	1600
25	$1666\frac{2}{3}$
30	1600

This area is maximum when $x = 25$ feet and $y = \frac{100}{3} = 33\frac{1}{3}$ feet.

—CONTINUED—

75. —CONTINUED—

(d) $A = \frac{8}{3}x(50 - x)$

$= -\frac{8}{3}(x^2 - 50x)$

$= -\frac{8}{3}(x^2 - 50x + 625 - 625)$

$= -\frac{8}{3}[(x - 25)^2 - 625]$

$= -\frac{8}{3}(x - 25)^2 + \frac{5000}{3}$

The maximum area occurs at the vertex and is 5000/3 square feet. This happens when $x = 25$ feet and $y = (200 - 4(25))/3 = 100/3$ feet. The dimensions are $2x = 50$ feet by $33\frac{1}{3}$ feet.

(e) They are all identical.

$x = 25$ feet and $y = 33\frac{1}{3}$ feet

76. (a) Radius of semicircular ends of track: $r = \frac{1}{2}y$

Distance around two semicircular parts of track:

$d = 2\pi r = 2\pi\left(\frac{1}{2}y\right) = \pi y$

(b) Distance traveled around track in one lap:

$d = \pi y + 2x = 200$

$\pi y = 200 - 2x$

$y = \frac{200 - 2x}{\pi}$

(c) Area of rectangular region:

$A = xy = x\left(\frac{200 - 2x}{\pi}\right)$

$= \frac{1}{\pi}(200x - 2x^2)$

$= -\frac{2}{\pi}(x^2 - 100x)$

$= -\frac{2}{\pi}(x^2 - 100x + 2500 - 2500)$

$= -\frac{2}{\pi}(x - 50)^2 + \frac{5000}{\pi}$

The area is maximum when $x = 50$ and

$y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}$.

77. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$

The vertex occurs at $-\frac{b}{2a} = \frac{-24/9}{2(-4/9)} = 3$. The maximum height is $y(3) = -\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12 = 16$ feet.

78. $y = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$

(a) The ball height when it is punted is the y-intercept.

$y = -\frac{16}{2025}(0)^2 + \frac{9}{5}(0) + 1.5 = 1.5$ feet

(b) The vertex occurs at $x = -\frac{b}{2a} = -\frac{9/5}{2(-16/2025)} = \frac{3645}{32}$.

The maximum height is $f\left(\frac{3645}{32}\right) = -\frac{16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + 1.5$

$$= -\frac{6561}{64} + \frac{6561}{32} + 1.5 = -\frac{6561}{64} + \frac{13,122}{64} + \frac{96}{64} = \frac{6657}{64} \text{ feet} \approx 104.02 \text{ feet.}$$

—CONTINUED—

78. —CONTINUED—

(c) The length of the punt is the positive x-intercept.

$$0 = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$$

$$x = \frac{-(9/5) \pm \sqrt{(9/5)^2 - (4)(1.5)(-16/2025)}}{-32/2025} \approx \frac{1.8 \pm 1.81312}{-0.01580247}$$

$x \approx -0.83031$ or $x \approx 228.64$

The punt is approximately 228.64 ft.

79. $C = 800 - 10x + 0.25x^2 = 0.25x^2 - 10x + 800$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{-10}{2(0.25)} = 20.$

The cost is minimum when $x = 20$ fixtures.

80. $C = 100{,}000 - 110x + 0.045x^2$

The vertex occurs at $x = -\frac{-110}{2(0.045)} \approx 1222.$

The cost is minimum when $x \approx 1222$ units.

81. $P = -0.0002x^2 + 140x - 250{,}000$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{140}{2(-0.0002)} = 350{,}000.$

The profit is maximum when $x = 350{,}000$ units.

82. $P = 230 + 20x - 0.5x^2$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{20}{2(-0.5)} = 20.$

Because x is in hundreds of dollars, $20 \times 100 = 2000$ dollars is the amount spent on advertising that gives maximum profit.

83. $R(p) = -25p^2 + 1200p$

(a) $R(20) = \$14{,}000$ thousand

$R(25) = \$14{,}375$ thousand

$R(30) = \$13{,}500$ thousand

(b) The revenue is a maximum at the vertex.

$$-\frac{b}{2a} = \frac{-1200}{2(-25)} = 24$$

$R(24) = 14{,}400$

The unit price that will yield a maximum revenue of \$14,400 thousand is \$24.

84. $R(p) = -12p^2 + 150p$

(a) $R(\$4) = -12(\$4)^2 + 150(\$4) = \$408$

$R(\$6) = -12(\$6)^2 + 150(\$6) = \$468$

$R(\$8) = -12(\$8)^2 + 150(\$8) = \$432$

(b) The vertex occurs at

$$p = -\frac{b}{2a} = -\frac{150}{2(-12)} = \$6.25$$

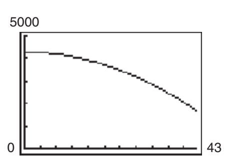
Revenue is maximum when price = \$6.25 per pet.

The maximum revenue is

$f(\$6.25) = -12(\$6.25)^2 + 150(\$6.25) = \$468.75.$

85. $C = 4299 - 1.8t - 1.36t^2, 0 \le t \le 43$

(a)



(b) Vertex $\approx (0, 4299)$

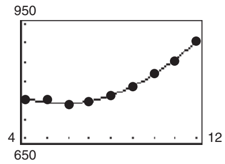
The vertex occurs when $y \approx 4299$ which is the maximum average annual consumption. The warnings may not have had an immediate effect, but over time they and other findings about the health risks and the increased cost of cigarettes have had an effect.

(c) $C(40) = 2051$

Annually: $\frac{209{,}128{,}094(2051)}{48{,}308{,}590} \approx 8879$ cigarettes

Daily: $\frac{8879}{366} \approx 24$ cigarettes

86. (a) and (c)



(b) $y = 4.303x^2 - 49.948x + 886.28$

(d) 1996

(e) Vertex occurs at

$$x = -\frac{b}{2a} = \frac{49.948}{2(4.303)} = 5.8$$

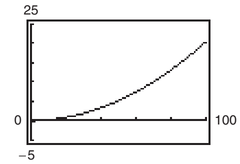
Minimum occurs at year ≈ 1996.

(f) $x = 18$

$y = 4.303(18)^2 - 49.948(18) + 886.28 = 1381.388$

There will be approximately 1,381,000 hairdressers and cosmetologists in 2008.

87. (a)



(b) $0.002s^2 + 0.005s - 0.029 = 10$

$$2s^2 + 5s - 29 = 10{,}000$$

$$2s^2 + 5s - 10{,}029 = 0$$

$a = 2, b = 5, c = -10{,}029$

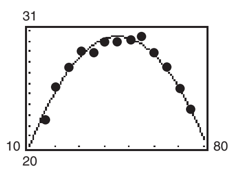
$$s = \frac{-5 \pm \sqrt{5^2 - 4(2)(-10{,}029)}}{2(2)}$$

$$s = \frac{-5 \pm \sqrt{80{,}257}}{4}$$

$s \approx -72.1, 69.6$

The maximum speed if power is not to exceed 10 horsepower is 69.6 miles per hour.

88. (a) and (c)



(b) $y = -0.0082x^2 + 0.746x + 13.47$

(d) The maximum of the graph is at $x \approx 45.5$, or about 45.5 mi/h. Algebraically, the maximum occurs at

$$x = -\frac{b}{2a} = \frac{-0.746}{2(-0.0082)} \approx 45.5 \text{ mi/h.}$$

89. True. The equation $-12x^2 - 1 = 0$ has no real solution, so the graph has no x-intercepts.

90. True. The vertex of $f(x)$ is $\left(-\frac{5}{4}, \frac{53}{4}\right)$ and the vertex of $g(x)$ is $\left(-\frac{5}{4}, -\frac{71}{4}\right)$.

91.
$$\begin{aligned}
f(x) &= ax^2 + bx + c \\
&= a\left(x^2 + \frac{b}{a}x\right) + c \\
&= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c \\
&= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c \\
&= a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + \frac{4ac - b^2}{4a}
\end{aligned}$$

$$\begin{aligned}
f\left(-\frac{b}{2a}\right) &= a\left(\frac{b^2}{4a^2}\right) + b\left(-\frac{b}{2a}\right) + c \\
&= \frac{b^2}{4a} - \frac{b^2}{2a} + c \\
&= \frac{b^2 - 2b^2 + 4ac}{4a} = \frac{4ac - b^2}{4a}
\end{aligned}$$

So, the vertex occurs at $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

92. Conditions (a) and (d) are preferable because profits would be increasing.

93. Yes. A graph of a quadratic equation whose vertex is (0, 0) has only one x-intercept.

94. If $f(x) = ax^2 + bx + c$ has two real zeros, then by the Quadratic Formula they are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The average of the zeros of f is

$$\frac{\dfrac{-b - \sqrt{b^2 - 4ac}}{2a} + \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}}{2} = \frac{\dfrac{-2b}{2a}}{2} = -\frac{b}{2a}.$$

This is the x-coordinate of the vertex of the graph.

95. $(-4, 3)$ and $(2, 1)$

$$m = \frac{1-3}{2-(-4)} = \frac{-2}{6} = -\frac{1}{3}$$

$$y - 1 = -\frac{1}{3}(x - 2)$$

$$y - 1 = -\frac{1}{3}x + \frac{2}{3}$$

$$y = -\frac{1}{3}x + \frac{5}{3}$$

96. $\left(\frac{7}{2}, 2\right)$, $m = \frac{3}{2}$

$$y - 2 = \frac{3}{2}\left(x - \frac{7}{2}\right)$$

$$y - 2 = \frac{3}{2}x - \frac{21}{4}$$

$$y = \frac{3}{2}x - \frac{13}{4}$$

97. $4x + 5y = 10 \implies y = -\frac{4}{5}x + 2$ and $m = -\frac{4}{5}$

The slope of the perpendicular line through $(0, 3)$ is $m = \frac{5}{4}$ and the y-intercept is $b = 3$.

$y = \frac{5}{4}x + 3$

98. $y = -3x + 2$

$m = -3$

For a parallel line, $m = -3$. So, for $(-8, 4)$, the line is

$$y - 4 = -3(x - (-8))$$

$$y - 4 = -3x - 24$$

$$y = -3x - 20.$$

For Exercises 99–104, let $f(x) = 14x - 3$, and $g(x) = 8x^2$.

99. $(f + g)(-3) = f(-3) + g(-3)$

$$= [14(-3) - 3] + 8(-3)^2 = 27$$

100. $(g - f)(2) = 8(2)^2 - 14(2) + 3 = 32 - 28 + 3 = 7$

101. $(fg)\left(-\frac{4}{7}\right) = f\left(-\frac{4}{7}\right)g\left(-\frac{4}{7}\right)$

$$= \left[14\left(-\frac{4}{7}\right) - 3\right]\left[8\left(-\frac{4}{7}\right)^2\right]$$

$$= (-11)\left(\frac{128}{49}\right) = -\frac{1408}{49}$$

102. $\left(\frac{f}{g}\right)(-1.5) = \frac{14(-1.5) - 3}{8(-1.5)^2} = \frac{-24}{18} = -\frac{4}{3}$

103. $(f \circ g)(-1) = f(g(-1)) = f(8) = 14(8) - 3 = 109$

104. $(g \circ f)(0) = g(f(0)) = g(14(0) - 3) = g(-3)$

$$= 8(-3)^2 = 72$$

105. Answers will vary.

Section 2.2 Polynomial Functions of Higher Degree

You should know the following basic principles about polynomials.

- $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_2x^2 + a_1x + a_0$, $a_n \neq 0$, is a polynomial function of degree n.
- If f is of odd degree and
  - (a) $a_n > 0$, then
    1. $f(x) \to \infty$ as $x \to \infty$.
    2. $f(x) \to -\infty$ as $x \to -\infty$.
  - (b) $a_n < 0$, then
    1. $f(x) \to -\infty$ as $x \to \infty$.
    2. $f(x) \to \infty$ as $x \to -\infty$.
- If f is of even degree and
  - (a) $a_n > 0$, then
    1. $f(x) \to \infty$ as $x \to \infty$.
    2. $f(x) \to \infty$ as $x \to -\infty$.
  - (b) $a_n < 0$, then
    1. $f(x) \to -\infty$ as $x \to \infty$.
    2. $f(x) \to -\infty$ as $x \to -\infty$.
- The following are equivalent for a polynomial function.
  - (a) $x = a$ is a zero of a function.
  - (b) $x = a$ is a solution of the polynomial equation $f(x) = 0$.
  - (c) $(x - a)$ is a factor of the polynomial.
  - (d) $(a, 0)$ is an x-intercept of the graph of f.
- A polynomial of degree n has at most n distinct zeros and at most $n - 1$ turning points.
- A factor $(x - a)^k$, $k > 1$, yields a repeated zero of $x = a$ of multiplicity k.
  - (a) If k is odd, the graph crosses the x-axis at $x = a$.
  - (b) If k is even, the graph just touches the x-axis at $x = a$.
- If f is a polynomial function such that $a < b$ and $f(a) \neq f(b)$, then f takes on every value between $f(a)$ and $f(b)$ in the interval $[a, b]$.
- If you can find a value where a polynomial is positive and another value where it is negative, then there is at least one real zero between the values.

Vocabulary Check

1. continuous

2. Leading Coefficient Test

3. n; $n - 1$

4. solution; $(x - a)$; x-intercept

5. touches; crosses

6. standard

7. Intermediate Value

1. $f(x) = -2x + 3$ is a line with y-intercept $(0, 3)$. Matches graph (c).

2. $f(x) = x^2 - 4x$ is a parabola with intercepts $(0, 0)$ and $(4, 0)$ and opens upward. Matches graph (g).

3. $f(x) = -2x^2 - 5x$ is a parabola with x-intercepts $(0, 0)$ and $\left(-\frac{5}{2}, 0\right)$ and opens downward. Matches graph (h).

4. $f(x) = 2x^3 - 3x + 1$ has intercepts $(0, 1)$, $(1, 0)$, $\left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}, 0\right)$ and $\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}, 0\right)$. Matches graph (f).

5. $f(x) = -\frac{1}{4}x^4 + 3x^2$ has intercepts $(0, 0)$ and $\left(\pm 2\sqrt{3}, 0\right)$. Matches graph (a).

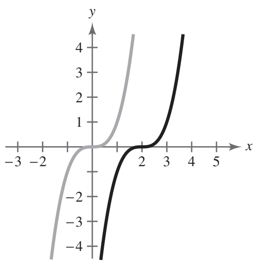
6. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$ has y-intercept $\left(0, -\frac{4}{3}\right)$. Matches graph (e).

7. $f(x) = x^4 + 2x^3$ has intercepts $(0, 0)$ and $(-2, 0)$. Matches graph (d).

8. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ has intercepts $(0, 0)$, $(1, 0)$, $(-1, 0)$, $(3, 0)$, $(-3, 0)$. Matches graph (b).

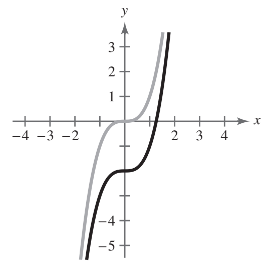
9. $y = x^3$

(a) $f(x) = (x - 2)^3$



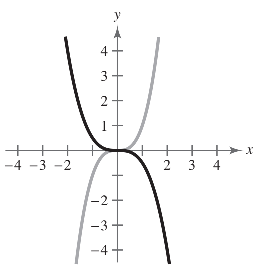
Horizontal shift two units to the right

(b) $f(x) = x^3 - 2$



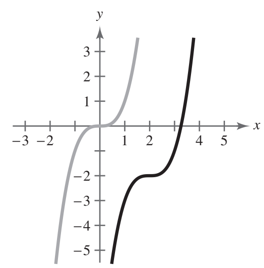
Vertical shift two units downward

(c) $f(x) = -\frac{1}{2}x^3$



Reflection in the x-axis and a vertical shrink

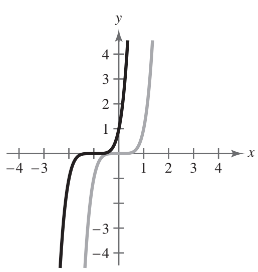
(d) $f(x) = (x - 2)^3 - 2$



Horizontal shift two units to the right and a vertical shift two units downward

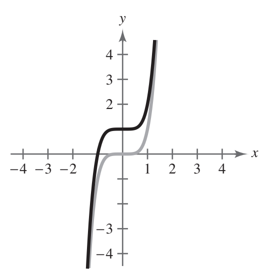
10. $y = x^5$

(a) $f(x) = (x + 1)^5$



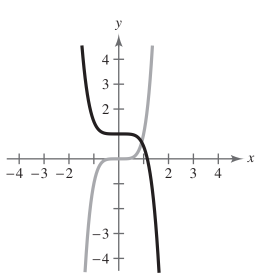
Horizontal shift one unit to the left

(b) $f(x) = x^5 + 1$



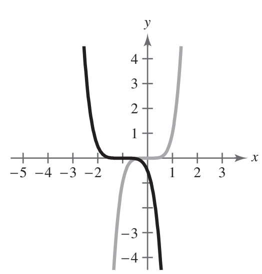
Vertical shift one unit upward

(c) $f(x) = 1 - \frac{1}{2}x^5$



Reflection in the x-axis, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), and vertical shift one unit upward

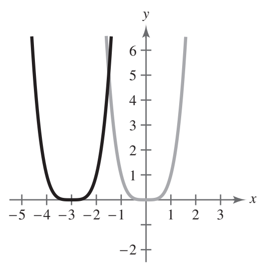
(d) $f(x) = -\frac{1}{2}(x + 1)^5$



Reflection in the x-axis, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), and horizontal shift one unit to the left

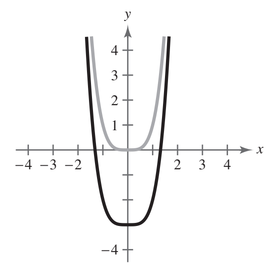
11. $y = x^4$

(a) $f(x) = (x + 3)^4$



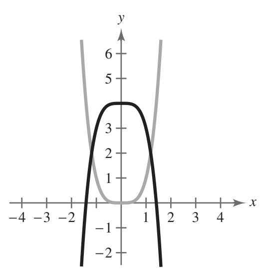
Horizontal shift three units to the left

(b) $f(x) = x^4 - 3$



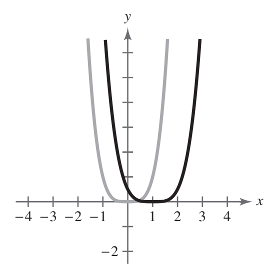
Vertical shift three units downward

(c) $f(x) = 4 - x^4$



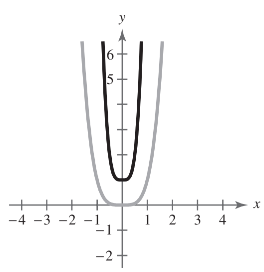
Reflection in the x-axis and then a vertical shift four units upward

(d) $f(x) = \frac{1}{2}(x - 1)^4$



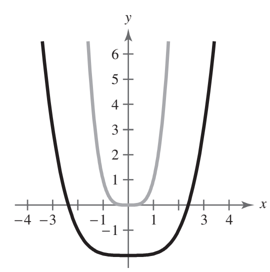
Horizontal shift one unit to the right and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

(e) $f(x) = (2x)^4 + 1$



Vertical shift one unit upward and a horizontal shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

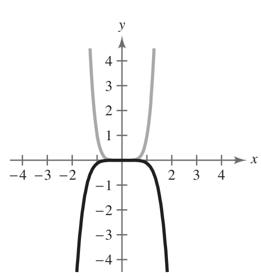
(f) $f(x) = \left(\frac{1}{2}x\right)^4 - 2$



Vertical shift two units downward and a horizontal stretch $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

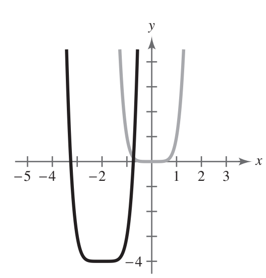
12. $y = x^6$

(a) $f(x) = -\frac{1}{8}x^6$



Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{8}\right)$ and reflection in the x-axis

(b) $f(x) = (x + 2)^6 - 4$

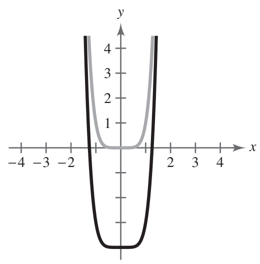


Horizontal shift two units to the left and vertical shift four units downward

—CONTINUED—

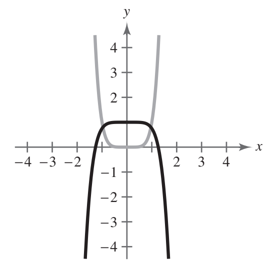
12. —CONTINUED—

(c) $f(x) = x^6 - 4$



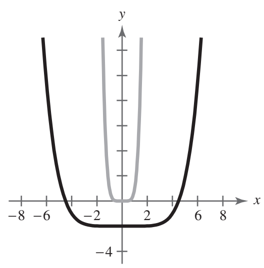
Vertical shift four units downward

(d) $f(x) = -\frac{1}{4}x^6 + 1$



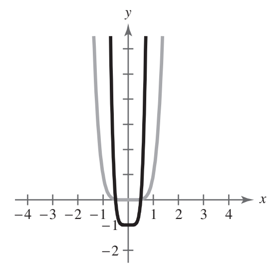
Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$, and vertical shift one unit upward

(e) $f(x) = \left(\frac{1}{4}x\right)^6 - 2$



Horizontal stretch (each x-value is multiplied by 4), and vertical shift two units downward

(f) $f(x) = (2x)^6 - 1$



Horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{2}\right)$, and vertical shift one unit downward

13. $f(x) = \frac{1}{3}x^3 + 5x$

Degree: 3

Leading coefficient: $\frac{1}{3}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

14. $f(x) = 2x^2 - 3x + 1$

Degree: 2

Leading coefficient: 2

The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

15. $g(x) = 5 - \frac{7}{2}x - 3x^2$

Degree: 2

Leading coefficient: -3

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

16. $h(x) = 1 - x^6$

Degree: 6

Leading coefficient: -1

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

17. $f(x) = -2.1x^5 + 4x^3 - 2$

Degree: 5

Leading coefficient: -2.1

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

18. $f(x) = 2x^5 - 5x + 7.5$

Degree: 5

Leading coefficient: 2

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

19. $f(x) = 6 - 2x + 4x^2 - 5x^3$

Degree: 3

Leading coefficient: -5

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

20. $f(x) = \dfrac{3x^4 - 2x + 5}{4}$

Degree: 4

Leading coefficient: $\frac{3}{4}$

The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

21. $h(t) = -\frac{2}{3}(t^2 - 5t + 3)$

Degree: 2

Leading coefficient: $-\frac{2}{3}$

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

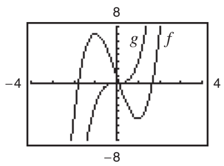
22. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

Degree: 3

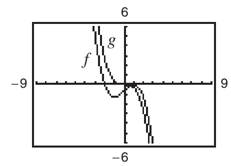
Leading coefficient: $-\frac{7}{8}$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

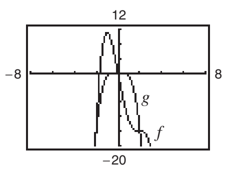
23. $f(x) = 3x^3 - 9x + 1;\ g(x) = 3x^3$



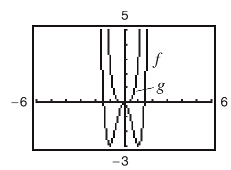
24. $f(x) = -\frac{1}{3}(x^3 - 3x + 2), g(x) = -\frac{1}{3}x^3$



25. $f(x) = -(x^4 - 4x^3 + 16x);\ g(x) = -x^4$



26. $f(x) = 3x^4 - 6x^2, g(x) = 3x^4$



27. $f(x) = x^2 - 25$

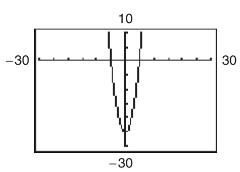
(a) $0 = x^2 - 25 = (x + 5)(x - 5)$

Zeros: $x = \pm 5$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning point: 1 (the vertex of the parabola)

(c)



28. (a) $f(x) = 49 - x^2$

$0 = (7 - x)(7 + x)$

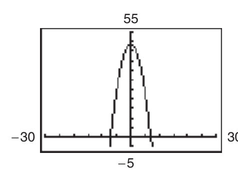
$x = \pm 7$, both with multiplicity 1

(b) Multiplicity of $x = 7$is 1.

Multiplicity of $x = -7$ is 1.

There is one turning point.

(c)



29. $h(t) = t^2 - 6t + 9$

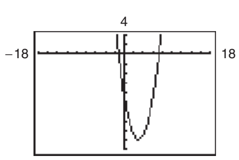
(a) $0 = t^2 - 6t + 9 = (t - 3)^2$

Zero: $t = 3$

(b) $t = 3$ has a multiplicity of 2 (even multiplicity).

Turning point: 1 (the vertex of the parabola)

(c)



30. (a) $f(x) = x^2 + 10x + 25$

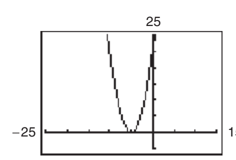
$0 = (x + 5)^2$

$x = -5$, with multiplicity 2

(b) The multiplicity of $x = -5$ is 2.

There is one turning point.

(c)



31. $f(x) = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$

(a) $0 = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$

$= \frac{1}{3}(x^2 + x - 2)$

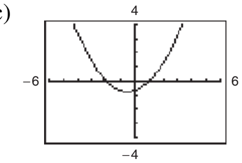
$= \frac{1}{3}(x + 2)(x - 1)$

Zeros: $x = -2, x = 1$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning point: 1 (the vertex of the parabola)

(c)



32. (a) $f(x) = \dfrac{1}{2}x^2 + \dfrac{5}{2}x - \dfrac{3}{2}$

$a = \dfrac{1}{2}, b = \dfrac{5}{2}, c = -\dfrac{3}{2}$

$$x = \frac{-\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4\left(\frac{1}{2}\right)\left(-\frac{3}{2}\right)}}{1}$$

$$= -\frac{5}{2} \pm \sqrt{\frac{37}{4}}$$

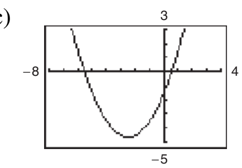
$= \dfrac{-5 \pm \sqrt{37}}{2}$, both with multiplicity 1

(b) The multiplicity of $\dfrac{-5 + \sqrt{37}}{2}$ is 1.

The multiplicity of $\dfrac{-5 + - \sqrt{37}}{2}$ is 1.

There is one turning point.

(c)



33. $f(x) = 3x^3 - 12x^2 + 3x$

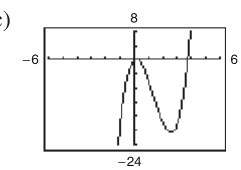
(a) $0 = 3x^3 - 12x^2 + 3x = 3x(x^2 - 4x + 1)$

Zeros: $x = 0, x = 2 \pm \sqrt{3}$ (by the Quadratic Formula)

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 2

(c)



34. (a) $g(x) = 5x(x^2 - 2x - 1)$

$0 = 5x(x^2 - 2x - 1)$

$0 = x(x^2 - 2x - 1)$

For $x^2 - 2x - 1$, $a = 1, b = -2, c = -1$.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= 1 \pm \sqrt{2}$$

The zeros are 0, $1 + \sqrt{2}$, and $1 - \sqrt{2}$, all with multiplicity 1.

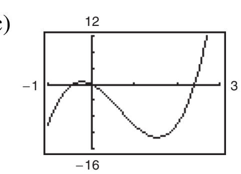
(b) The multiplicity of $x = 0$ is 1.

The multiplicity of $x = 1 + \sqrt{2}$ is 1.

The multiplicity of $x = 1 - \sqrt{2}$ is 1.

There are two turning points.

(c)



35. $f(t) = t^3 - 4t^2 + 4t$

(a) $0 = t^3 - 4t^2 + 4t = t(t^2 - 4t + 4) = t(t - 2)^2$

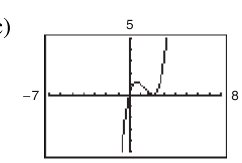
Zeros: $t = 0, t = 2$

(b) $t = 0$ has a multiplicity of 1 (odd multiplicity).

$t = 2$ has a multiplicity of 2 (even multiplicity).

Turning points: 2

(c)



36. (a) $f(x) = x^4 - x^3 - 20x^2$

$0 = x^2(x^2 - x - 20)$

$0 = x^2(x + 4)(x - 5)$

$x = 0, -4, 5$

0 with multiplicity 2, -4 and 5 with multiplicity 1.

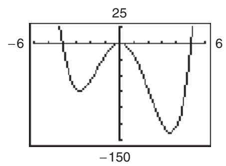
(b) The multiplicity of $x = 0$ is 2.

The multiplicity of $x = 5$ is 1.

The multiplicity of $x = -4$ is 1.

There are three turning points.

(c)



37. $g(t) = t^5 - 6t^3 + 9t$

(a) $0 = t^5 - 6t^3 + 9t = t(t^4 - 6t^2 + 9) = t(t^2 - 3)^2$

$= t\left(t + \sqrt{3}\right)^2\left(t - \sqrt{3}\right)^2$

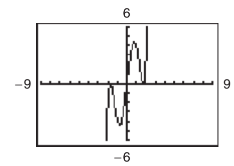
Zeros: $t = 0$, $t = \pm\sqrt{3}$

(b) $t = 0$ has a multiplicity of 1 (odd multiplicity).

$t = \pm\sqrt{3}$ each have a multiplicity of 2 (even multiplicity).

Turning points: 4

(c)



38. (a) $f(x) = x^5 + x^3 - 6x$

$0 = x(x^4 + x^2 - 6)$

$0 = x(x^2 + 3)(x^2 - 2)$

$x = 0, \pm\sqrt{2}$, all with multiplicity 1

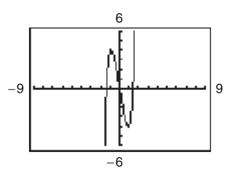
(b) The multiplicity of $x = 0$ is 1.

The multiplicity of $x = \sqrt{2}$ is 1.

The multiplicity of $x = -\sqrt{2}$ is 1.

There are two turning points.

(c)



39. $f(x) = 5x^4 + 15x^2 + 10$

(a) $0 = 5x^4 + 15x^2 + 10$

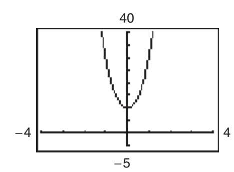
$= 5(x^4 + 3x^2 + 2)$

$= 5(x^2 + 1)(x^2 + 2)$

No real zeros

(b) Turning point: 1

(c)



40. (a) $f(x) = 2x^4 - 2x^2 - 40$

$0 = 2x^4 - 2x^2 - 40$

$0 = 2(x^2 + 4)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)$

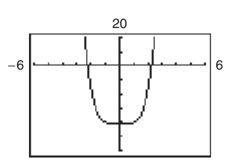
$x = \pm\sqrt{5}$, both with multiplicity 1

(b) The multiplicity of $x = \sqrt{5}$ is 1.

The multiplicity of $x = -\sqrt{5}$ is 1.

There is one turning point.

(c)



41. $g(x) = x^3 + 3x^2 - 4x - 12$

(a) $0 = x^3 + 3x^2 - 4x - 12 = x^2(x + 3) - 4(x + 3)$

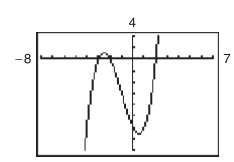
$= (x^2 - 4)(x + 3) = (x - 2)(x + 2)(x + 3)$

Zeros: $x = \pm 2$, $x = -3$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 2

(c)



42. (a) $f(x) = x^3 - 4x^2 - 25x + 100$

$0 = x^2(x - 4) - 25(x - 4)$

$0 = (x^2 - 25)(x - 4)$

$0 = (x + 5)(x - 5)(x - 4)$

$x = \pm 5, 4$, all with multiplicity 1

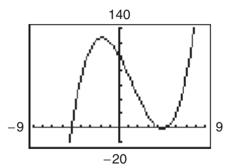
(b) The multiplicity of $x = 5$ is 1.

The multiplicity of $x = -5$ is 1.

The multiplicity of $x = 4$ is 1.

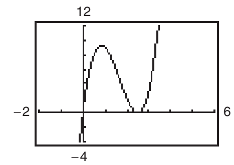
There are two turning points.

(c)



43. $y = 4x^3 - 20x^2 + 25x$

(a)



(b) x-intercepts: $(0, 0), \left(\frac{5}{2}, 0\right)$

(c) $0 = 4x^3 - 20x^2 + 25x$

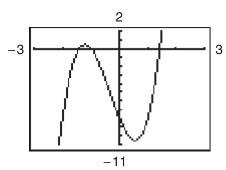
$0 = x(2x - 5)^2$

$x = 0$ or $x = \frac{5}{2}$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

44. $y = 4x^3 + 4x^2 - 8x - 8$

(a)



(b) $(-1, 0), (-1.414214, 0), (1.414214, 0)$

(c) $0 = 4x^3 + 4x^2 - 8x - 8$

$0 = 4x^2(x + 1) - 8(x + 1)$

$0 = (4x^2 - 8)(x + 1)$

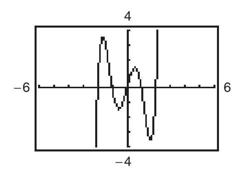
$0 = 4(x^2 - 2)(x + 1)$

$x = \pm\sqrt{2}, -1$

(d) The intercepts match part (b).

45. $y = x^5 - 5x^3 + 4x$

(a)



(b) x-intercepts: $(0, 0), (\pm 1, 0), (\pm 2, 0)$

(c) $0 = x^5 - 5x^3 + 4x$

$0 = x(x^2 - 1)(x^2 - 4)$

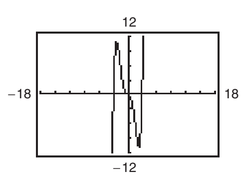
$0 = x(x + 1)(x - 1)(x + 2)(x - 2)$

$x = 0, \pm 1, \pm 2$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

46. $y = \frac{1}{4}x^3(x^2 - 9)$

(a)



(b) $(0, 0), (3, 0), (-3, 0)$

(c) $0 = \frac{1}{4}x^3(x^2 - 9)$

$x = 0, \pm 3$

x-intercepts: $(0, 0), (\pm 3, 0)$

(d) The intercepts match part (b).

47. $f(x) = (x - 0)(x - 10)$

$f(x) = x^2 - 10x$

Note: $f(x) = a(x - 0)(x - 10) = ax(x - 10)$ has zeros 0 and 10 for all real numbers $a \neq 0$.

48. $f(x) = (x - 0)(x - (-3))$

$= x(x + 3)$

$= x^2 + 3x$

Note: $f(x) = ax(x + 3)$ has zeros 0 and -3 for all real numbers a.

49. $f(x) = (x - 2)(x - (-6))$
$= (x - 2)(x + 6)$
$= x^2 + 4x - 12$

Note: $f(x) = a(x - 2)(x + 6)$ has zeros 2 and -6 for all real numbers $a \neq 0$.

50. $f(x) = (x - (-4))(x - 5)$
$= (x + 4)(x - 5)$
$= x^2 - x - 20$

Note: $f(x) = a(x + 4)(x - 5)$ has zeros -4 and 5 for all real numbers a.

51. $f(x) = (x - 0)(x - (-2))(x - (-3))$
$= x(x + 2)(x + 3)$
$= x^3 + 5x^2 + 6x$

Note: $f(x) = ax(x + 2)(x + 3)$ has zeros $0, -2, -3$ for all real numbers $a \neq 0$.

52. $f(x) = (x - 0)(x - 2)(x - 5)$
$= x(x - 2)(x - 5)$
$= x(x^2 - 7x + 10)$
$= x^3 - 7x^2 + 10x$

Note: $f(x) = ax(x - 2)(x - 5)$ has zeros 0, 2, 5 for all real numbers a.

53. $f(x) = (x - 4)(x + 3)(x - 3)(x - 0)$
$= (x - 4)(x^2 - 9)x$
$= x^4 - 4x^3 - 9x^2 + 36x$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has these zeros for all real numbers $a \neq 0$.

54. $f(x) = \left(x - (-2)\right)\left(x - (-1)\right)(x - 0)(x - 1)(x - 2)$
$= x(x + 2)(x + 1)(x - 1)(x - 2)$
$= x(x^2 - 4)(x^2 - 1)$
$= x(x^4 - 5x^2 + 4)$
$= x^5 - 5x^3 + 4x$

Note: $f(x) = ax(x + 2)(x + 1)(x - 1)(x - 2)$ has zeros $-2, -1, 0, 1, 2$ for all real numbers a.

55. $f(x) = \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right]$
$= \left[(x - 1) - \sqrt{3}\right]\left[(x - 1) + \sqrt{3}\right]$
$= (x - 1)^2 - \left(\sqrt{3}\right)^2$
$= x^2 - 2x + 1 - 3$
$= x^2 - 2x - 2$

Note: $f(x) = a(x^2 - 2x - 2)$ has these zeros for all real numbers $a \neq 0$.

56. $f(x) = (x - 2)\left[x - \left(4 + \sqrt{5}\right)\right]\left[x - \left(4 - \sqrt{5}\right)\right]$
$= (x - 2)\left[(x - 4) - \sqrt{5}\right]\left[(x - 4) + \sqrt{5}\right]$
$= (x - 2)\left[(x - 4)^2 - 5\right]$
$= x(x - 4)^2 - 5x - 2(x - 4)^2 + 10$
$= x^3 - 8x^2 + 16x - 5x - 2x^2 + 16x - 32 + 10$
$= x^3 - 10x^2 + 27x - 22$

Note: $f(x) = a(x^3 - 10x^2 + 27x - 22)$ has these zeros for all real numbers a.

57. $f(x) = (x - (-2))(x - (-2))$
$= (x + 2)^2 = x^2 + 4x + 4$

Note: $f(x) = a(x^2 + 4x + 4)$, $a \neq 0$, has degree 2 and zero $x = -2$.

58. $f(x) = [x - (-8)][x - (-4)]$
$= (x + 8)(x + 4) = x^2 + 12x + 32$

Note: $f(x) = a(x^2 + 12x + 32)$, $a \neq 0$, has degree 2 and zeros $x = -8$ and -4.

59. $f(x) = (x - (-3))(x - 0)(x - 1)$
$= x(x + 3)(x - 1) = x^3 + 2x^2 - 3x$

Note: $f(x) = a(x^3 + 2x^2 - 3x)$, $a \neq 0$, has degree 3 and zeros $x = -3, 0, 1$.

60. $f(x) = (x + 2)(x - 4)(x - 7)$
$= (x + 2)(x^2 - 11x + 28) = x^3 - 9x^2 + 6x + 56$

Note: $f(x) = a(x^3 - 9x^2 + 6x + 56)$, $a \neq 0$, has degree 3 and zeros $x = -2, 4$, and 7.

61. $f(x) = (x - 0)\left(x - \sqrt{3}\right)\left(x - \left(-\sqrt{3}\right)\right)$
$= x\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) = x^3 - 3x$

Note: $f(x) = a(x^3 - 3x)$, $a \neq 0$, has degree 3 and zeros $x = 0, \sqrt{3}, -\sqrt{3}$.

62. $f(x) = (x - 9)^3 = x^3 - 27x^2 + 243x - 729$

Note: $f(x) = a(x^3 - 27x^2 + 243x - 729)$, $a \neq 0$, has degree 3 and zero $x = 9$.

63. $f(x) = (x - (-5))^2(x - 1)(x - 2) = x^4 + 7x^3 - 3x^2 - 55x + 50$

or $f(x) = (x - (-5))(x - 1)^2(x - 2) = x^4 + x^3 - 15x^2 + 23x - 10$

or $f(x) = (x - (-5))(x - 1)(x - 2)^2 = x^4 - 17x^2 + 36x - 20$

Note: Any nonzero scalar multiple of these functions would also have degree 4 and zeros $x = -5, 1, 2$.

64. $f(x) = (x + 4)(x + 1)(x - 3)(x - 6) = x^4 - 4x^3 - 23x^2 + 54x + 72$

Note: $f(x) = a(x^4 - 4x^3 - 23x^2 + 54x + 72)$, $a \neq 0$, has degree 4 and zeros $x = -4, -1, 3$, and 6.

65. $f(x) = x^4(x + 4) = x^5 + 4x^4$

or $f(x) = x^3(x + 4)^2 = x^5 + 8x^4 + 16x^3$

or $f(x) = x^2(x + 4)^3 = x^5 + 12x^4 + 48x^3 + 64x^2$

or $f(x) = x(x + 4)^4 = x^5 + 16x^4 + 96x^3 + 256x^2 + 256x$

Note: Any nonzero scalar multiple of these functions would also have degree 5 and zeros $x = 0$ and -4.

66. $f(x) = (x + 3)^2(x - 1)(x - 5)(x - 6) = x^5 - 6x^4 - 22x^3 + 108x^2 + 189x - 270$

or $f(x) = (x + 3)(x - 1)^2(x - 5)(x - 6) = x^5 - 10x^4 + 14x^3 + 88x^2 - 183x + 90$

or $f(x) = (x + 3)(x - 1)(x - 5)^2(x - 6) = x^5 - 14x^4 + 50x^3 + 68x^2 - 555x + 450$

or $f(x) = (x + 3)(x - 1)(x - 5)(x - 6)^2 = x^5 - 15x^4 + 59x^3 + 63x^2 - 648x + 540$

Note: Any nonzero multiple of these functions would also have degree 5 and zeros $x = -3, 1, 5$, and 6.

67. $f(x) = x^3 - 9x = x(x^2 - 9) = x(x + 3)(x - 3)$

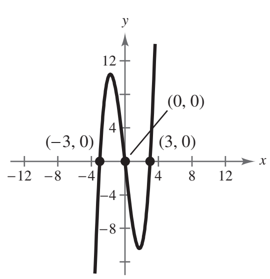
(a) Falls to the left; rises to the right

(b) Zeros: $0, -3, 3$

(c)

x	-3	-2	-1	0	1	2	3
$f(x)$	0	10	8	0	-8	-10	0

(d)



68. $g(x) = x^4 - 4x^2 = x^2(x + 2)(x - 2)$

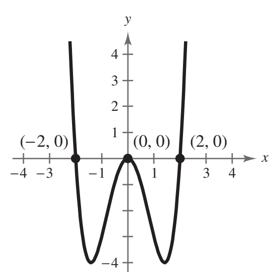
(a) Rises to the left; rises to the right

(b) Zeros: $-2, 0, 2$

(c)

x	± 0.5	± 1	± 1.5	± 2.5
$g(x)$	-0.94	-3	-3.94	14.1

(d)



69. $f(t) = \frac{1}{4}(t^2 - 2t + 15) = \frac{1}{4}(t - 1)^2 + \frac{7}{2}$

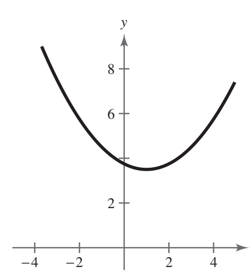
(a) Rises to the left; rises to the right

(b) No real zero (no x-intercepts)

(c)

t	-1	0	1	2	3
$f(t)$	4.5	3.75	3.5	3.75	4.5

(d) The graph is a parabola with vertex $\left(1, \frac{7}{2}\right)$.



70. $g(x) = -x^2 + 10x - 16 = -(x - 2)(x - 8)$

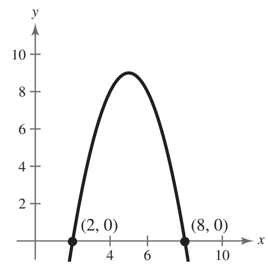
(a) Falls to the left; falls to the right

(b) Zeros: 2, 8

(c)

x	1	3	5	7	9
$g(x)$	-7	5	9	5	-7

(d)



71. $f(x) = x^3 - 3x^2 = x^2(x - 3)$

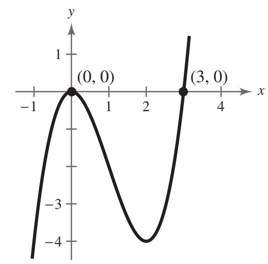
(a) Falls to the left; rises to the right

(b) Zeros: 0, 3

(c)

x	-1	0	1	2	3
$f(x)$	-4	0	-2	-4	0

(d)



72. $f(x) = 1 - x^3$

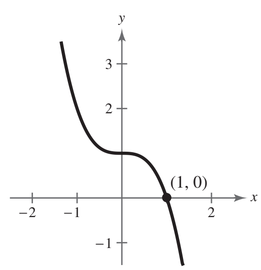
(a) Rises to the left; falls to the right

(b) Zero: 1

(c)

x	-2	-1	0	1	2
$f(x)$	9	2	1	0	-7

(d)



73. $f(x) = 3x^3 - 15x^2 + 18x = 3x(x - 2)(x - 3)$

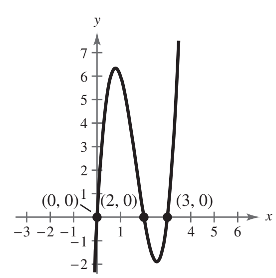
(a) Falls to the left; rises to the right

(b) Zeros: 0, 2, 3

(c)

x	0	1	2	2.5	3	3.5
$f(x)$	0	6	0	-1.875	0	7.875

(d)



74. $f(x) = -4x^3 + 4x^2 + 15x$

$= -x(4x^2 - 4x - 15)$

$= -x(2x - 5)(2x + 3)$

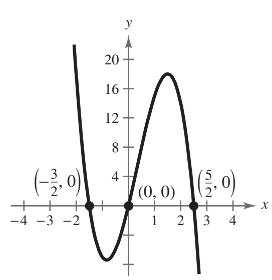
(a) Rises to the left; falls to the right

(b) Zeros: $-\frac{3}{2}, 0, \frac{5}{2}$

(c)

x	-3	-2	-1	0	1	2	3
$f(x)$	99	18	-7	0	15	14	-27

(d)



75. $f(x) = -5x^2 - x^3 = -x^2(5 + x)$

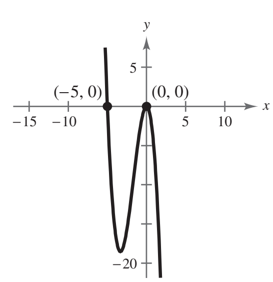
(a) Rises to the left; falls to the right

(b) Zeros: $0, -5$

(c)

x	-5	-4	-3	-2	-1	0	1
$f(x)$	0	-16	-18	-12	-4	0	-6

(d)



76. $f(x) = -48x^2 + 3x^4$

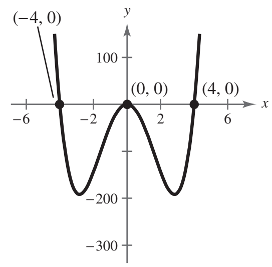
$= 3x^2(x^2 - 16)$

(a) Rises to the left; rises to the right

(b) Zeros: $0, \pm 4$

(c)

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	675	0	-189	-144	-45	0	-45	-144	-189	0	675

(d) 

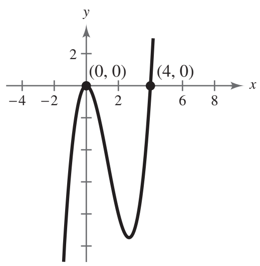
77. $f(x) = x^2(x - 4)$

(a) Falls to the left; rises to the right

(b) Zeros: $0, 4$

(c)

x	-1	0	1	2	3	4	5
$f(x)$	-5	0	-3	-8	-9	0	25

(d) 

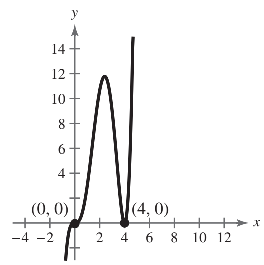
78. $h(x) = \frac{1}{3}x^3(x - 4)^2$

(a) Falls to the left; rises to the right

(b) Zeros: $0, 4$

(c)

x	-1	0	1	2	3	4	5
$h(x)$	$-\frac{25}{3}$	0	3	$\frac{32}{3}$	9	0	$\frac{125}{3}$

(d) 

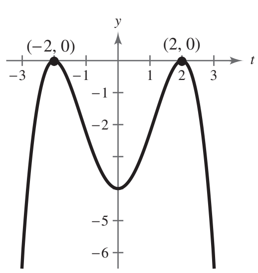
79. $g(t) = -\frac{1}{4}(t - 2)^2(t + 2)^2$

(a) Falls to the left; falls to the right

(b) Zeros: $2, -2$

(c)

t	-3	-2	-1	0	1	2	3
$g(t)$	$-\frac{25}{4}$	0	$-\frac{9}{4}$	-4	$-\frac{9}{4}$	0	$-\frac{25}{4}$

(d) 

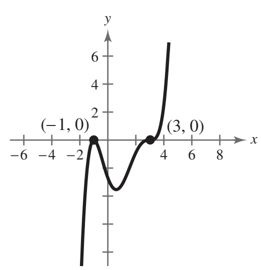
80. $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^3$

(a) Falls to the left; rises to the right

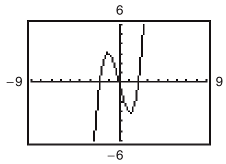
(b) Zeros: $-1, 3$

(c)

x	-2	-1	0	1	2	4
$g(x)$	-12.5	0	-2.7	-3.2	-0.9	2.5

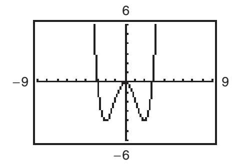
(d) 

81. $f(x) = x^3 - 4x = x(x + 2)(x - 2)$



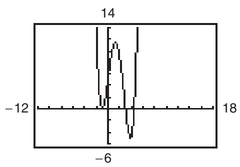
Zeros: $0, -2, 2$ all of multiplicity 1

82. $f(x) = \frac{1}{4}x^4 - 2x^2$



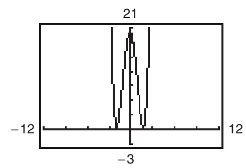
Zeros: -2.828 and 2.828 with multiplicity 1; 0, with multiplicity 2

83. $g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$



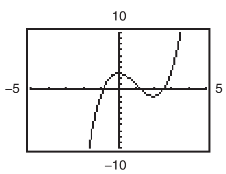
Zeros: -1 of multiplicity 2; 3 of multiplicity 1; $\frac{9}{2}$ of multiplicity 1

84. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$



Zeros: $-2, \frac{5}{3}$, both with multiplicity 2

85. $f(x) = x^3 - 3x^2 + 3$

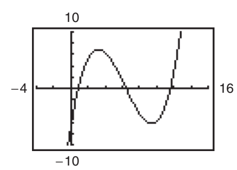


The function has three zeros. They are in the intervals $(-1, 0)$, $(1, 2)$ and $(2, 3)$. They are $x \approx -0.879, 1.347, 2.532$.

x	y_1
-3	-51
-2	-17
-1	-1
0	3
1	1
2	-1
3	3
4	19

86. $f(x) = 0.11x^3 - 2.07x^2 + 9.81x - 6.88$

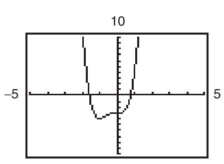
The function has three zeros. They are in the intervals $(0, 1)$, $(6, 7)$, and $(11, 12)$. They are approximately 0.845, 6.385, and 11.588.



x	y
0	-6.88
1	0.97
2	5.34
3	6.89
4	6.28
5	4.17
6	1.12

x	y
7	-1.91
8	-4.56
9	-6.07
10	-5.78
11	-3.03
12	2.84

87. $g(x) = 3x^4 + 4x^3 - 3$

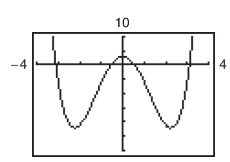


The function has two zeros. They are in the intervals $(-2, -1)$ and $(0, 1)$. They are $x \approx -1.585, 0.779$.

x	y_1
-4	509
-3	132
-2	13
-1	-4
0	-3
1	4
2	77
3	348

88. $h(x) = x^4 - 10x^2 + 3$

The function has four zeros. They are in the intervals $(-4, -3)$, $(-1, 0)$, $(0, 1)$, and $(3, 4)$. They are approximately ± 3.113 and ± 0.556.



x	y
-4	99
-3	-6
-2	-21
-1	-6
0	3
1	-6
2	-21
3	-6
4	99

89. (a) Volume $= l \cdot w \cdot h$

height $= x$

length $=$ width $= 36 - 2x$

Thus, $V(x) = (36 - 2x)(36 - 2x)(x) = x(36 - 2x)^2$.

(b) Domain: $0 < x < 18$

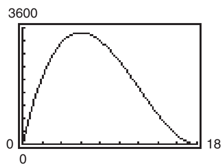
The length and width must be positive.

(c)

Box Height	Box Width	Box Volume, V
1	$36 - 2(1)$	$1[36 - 2(1)]^2 = 1156$
2	$36 - 2(2)$	$2[36 - 2(2)]^2 = 2048$
3	$36 - 2(3)$	$3[36 - 2(3)]^2 = 2700$
4	$36 - 2(4)$	$4[36 - 2(4)]^2 = 3136$
5	$36 - 2(5)$	$5[36 - 2(5)]^2 = 3380$
6	$36 - 2(6)$	$6[36 - 2(6)]^2 = 3456$
7	$36 - 2(7)$	$7[36 - 2(7)]^2 = 3388$

The volume is a maximum of 3456 cubic inches when the height is 6 inches and the length and width are each 24 inches. So the dimensions are $6 \times 24 \times 24$ inches.

(d)



The maximum point on the graph occurs at $x = 6$. This agrees with the maximum found in part (c).

90. (a) Volume $= l \cdot w \cdot h = (24 - 2x)(24 - 4x)x$

$= 2(12 - x) \cdot 4(6 - x)x$

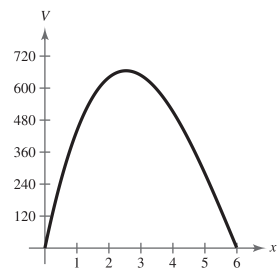
$= 8x(12 - x)(6 - x)$

(b) $x > 0, \quad 12 - x > 0, \quad 6 - x > 0$

$x < 12 \qquad x < 6$

Domain: $0 < x < 6$

(c)



$x \approx 2.6$ corresponds to a maximum of about 665 cubic inches.

91. (a) $A = l \cdot w = (12 - 2x)(x) = -2x^2 + 12x$ square inches

(b) 16 feet = 192 inches

$V = l \cdot w \cdot h$

$= (12 - 2x)(x)(192)$

$= -384x^2 + 2304x$ cubic inches

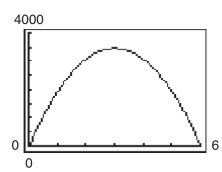
(c) Since x and $12 - 2x$ cannot be negative, we have $0 < x < 6$ inches for the domain.

(d)

x	V
0	0
1	1920
2	3072
3	3456
4	3072
5	1920
6	0

When $x = 3$, the volume is a maximum with $V = 3456$ in.^{3}. The dimensions of the gutter cross-section are 3 inches $\times$ 6 inches $\times$ 3 inches.

(e)



Maximum: $(3, 3456)$

The maximum value is the same.

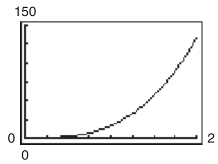
(f) No. The volume is a product of the constant length and the cross-sectional area. The value of x would remain the same; only the value of V would change if the length was changed.

92. (a) $V = \frac{4}{3}\pi r^3 + \pi r^2(4r)$

$V = \frac{4}{3}\pi r^3 + 4\pi r^3$

$= \frac{16}{3}\pi r^3$

(b) $r \geq 0$

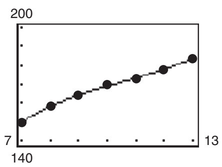
(c) 

(d) $V = 120 \text{ ft}^3 = \frac{16}{3}\pi r^3$

$r = 1.93$ ft

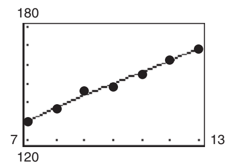
length $= 4r = 7.72$ ft

93. $y_1 = 0.139t^3 - 4.42t^2 + 51.1t - 39$



The model is a good fit to the actual data.

94. $y = 0.056t^3 - 1.73t^2 + 23.8t + 29$



The data fit the model closely.

95. Midwest: $y_1(18) = \$259.368$ thousand $= \$259{,}368$

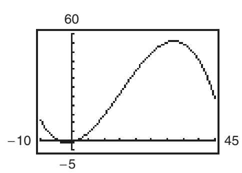
South: $y_2(18) = \$223.472$ thousand $= \$223{,}472$

Since the models are both cubic functions with positive leading coefficients, both will increase without bound as t increases, thus should only be used for short term projections.

96. Answers will vary.

Example: The median price of homes in the South are all lower than those in the Midwest. The curves do not intersect.

97. $G = -0.003t^3 + 0.137t^2 + 0.458t - 0.839, \; 2 \leq t \leq 34$

(a) 

(b) The tree is growing most rapidly at $t \approx 15$.

(c) $y = -0.009t^2 + 0.274t + 0.458$

$$-\frac{b}{2a} = \frac{-0.274}{2(-0.009)} \approx 15.222$$

$y(15.222) \approx 2.543$

Vertex $\approx (15.22, 2.54)$

(d) The x-value of the vertex in part (c) is approximately equal to the value found in part (b).

98. $R = \frac{1}{100{,}000}(-x^3 + 600x^2)$

The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$. The point is $(200, 160)$ which corresponds to spending \$2,000,000 on advertising to obtain a revenue of \$160 million.

99. False. A fifth degree polynomial can have at most four turning points.

100. True. $f(x) = (x - 1)^6$ has one repeated solution.

101. True. A polynomial of degree 7 with a negative leading coefficient rises to the left and falls to the right.

102. (a) Degree: 3
Leading coefficient: Positive

(b) Degree: 2
Leading coefficient: Positive

(c) Degree: 4
Leading coefficient: Positive

(d) Degree: 5
Leading coefficient: Positive

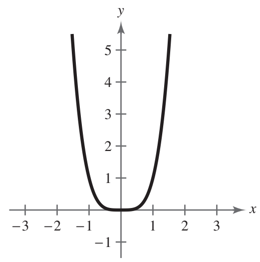
103. $f(x) = x^4$; $f(x)$ is even.

(a) $g(x) = f(x) + 2$

Vertical shift two units upward

$$g(-x) = f(-x) + 2$$
$$= f(x) + 2$$
$$= g(x)$$

Even



(b) $g(x) = f(x + 2)$

Horizontal shift two units to the left

Neither odd nor even

(c) $g(x) = f(-x) = (-x)^4 = x^4$

Reflection in the y-axis. The graph looks the same.

Even

(d) $g(x) = -f(x) = -x^4$

Reflection in the x-axis

Even

(e) $g(x) = f\left(\frac{1}{2}x\right) = \frac{1}{16}x^4$

Horizontal stretch

Even

(f) $g(x) = \frac{1}{2}f(x) = \frac{1}{2}x^4$

Vertical shrink

Even

(g) $g(x) = f(x^{3/4}) = (x^{3/4})^4 = x^3, x \geq 0$

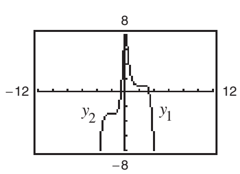
Neither odd nor even

(h) $g(x) = (f \circ f)(x) = f(f(x)) = f(x^4) = (x^4)^4 = x^{16}$

Even

104. (a) $y_1 = -\frac{1}{3}(x - 2)^5 + 1$ is decreasing.

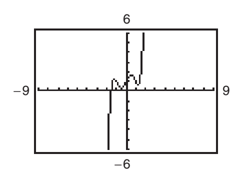
$y_2 = \frac{3}{5}(x + 2)^5 - 3$ is increasing.



(b) The graph is either always increasing or always decreasing. The behavior is determined by a. If $a > 0$, $g(x)$ will always be increasing. If $a < 0$, $g(x)$ will always be decreasing.

(c) $H(x) = x^5 - 3x^3 + 2x + 1$

Since $H(x)$ is not always increasing or always decreasing, $H(x)$ cannot be written in the form $a(x - h)^5 + k$.



105. $5x^2 + 7x - 24 = (5x - 8)(x + 3)$

106. $6x^3 - 61x^2 + 10x = x(6x^2 - 61x + 10)$
$$= x(6x - 1)(x - 10)$$

107. $4x^4 - 7x^3 - 15x^2 = x^2(4x^2 - 7x - 15)$
$$= x^2(4x + 5)(x - 3)$$

108. $y^3 + 216 = y^3 + 6^3$
$$= (y + 6)(y^2 - 6y + 36)$$

109. $2x^2 - x - 28 = 0$

$(2x + 7)(x - 4) = 0$

$2x + 7 = 0 \implies x = -\frac{7}{2}$

$x - 4 = 0 \implies x = 4$

110. $3x^2 - 22x - 16 = 0$

$(3x + 2)(x - 8) = 0$

$3x + 2 = 0$ or $x - 8 = 0$

$x = -\frac{2}{3}$ or $x = 8$

111. $12x^2 + 11x - 5 = 0$

$$(3x - 1)(4x + 5) = 0$$
$$3x - 1 = 0 \Rightarrow x = \tfrac{1}{3}$$
$$4x + 5 = 0 \Rightarrow x = -\tfrac{5}{4}$$

112. $x^2 + 24x + 144 = 0$

$$(x + 12)^2 = 0$$
$$x + 12 = 0$$
$$x = -12$$

113. $x^2 - 2x - 21 = 0$

$$(x^2 - 2x + (-1)^2) - 21 - 1 = 0$$
$$(x - 1)^2 - 22 = 0$$
$$(x - 1)^2 = 22$$
$$x - 1 = \pm\sqrt{22}$$
$$x = 1 \pm \sqrt{22}$$

114. $x^2 - 8x + 2 = 0$

$$x^2 - 8x = -2$$
$$x^2 - 8x + 16 = -2 + 16$$
$$(x - 4)^2 = 14$$
$$x - 4 = \pm\sqrt{14}$$
$$x = 4 \pm \sqrt{14}$$

115. $2x^2 + 5x - 20 = 0$

$$2\left(x^2 + \frac{5}{2}x\right) - 20 = 0$$
$$2\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2\right) - 20 - \frac{25}{8} = 0$$
$$2\left(x + \frac{5}{4}\right)^2 - \frac{185}{8} = 0$$
$$\left(x + \frac{5}{4}\right)^2 = \frac{185}{16}$$
$$x + \frac{5}{4} = \pm\frac{\sqrt{185}}{4}$$
$$x = \frac{-5 \pm \sqrt{185}}{4}$$

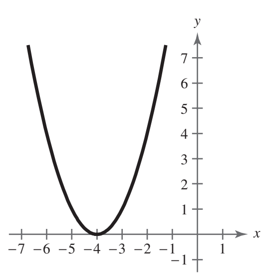
116. $3x^2 + 4x - 9 = 0$

$$x^2 + \frac{4}{3}x - 3 = 0$$
$$x^2 + \frac{4}{3}x = 3$$
$$x^2 + \frac{4}{3}x + \frac{4}{9} = 3 + \frac{4}{9}$$
$$\left(x + \frac{2}{3}\right)^2 = \frac{31}{9}$$
$$x + \frac{2}{3} = \pm\sqrt{\frac{31}{9}}$$
$$x = -\frac{2}{3} \pm \frac{\sqrt{31}}{3}$$
$$x = \frac{-2 \pm \sqrt{31}}{3}$$

117. $f(x) = (x + 4)^2$

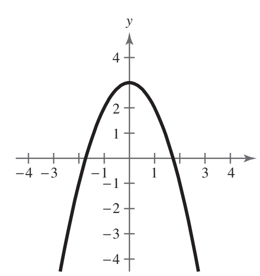
Common function: $y = x^2$

Transformation: Horizontal shift four units to the left



118. $f(x) = 3 - x^2$

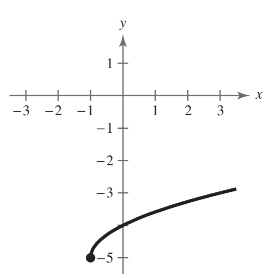
Reflection in the x-axis and vertical shift of three units upward of $y = x^2$



119. $f(x) = \sqrt{x + 1} - 5$

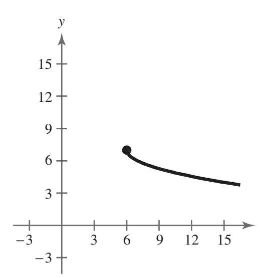
Common function: $y = \sqrt{x}$

Transformation: Horizontal shift one unit to the left and a vertical shift five units downward



120. $f(x) = 7 - \sqrt{x - 6}$

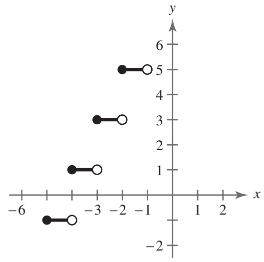
Horizontal shift of six units to the right, reflection in the x-axis, and vertical shift of seven units upward of $y = \sqrt{x}$



121. $f(x) = 2[\![x]\!] + 9$

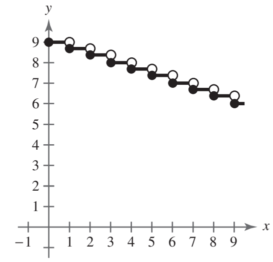
Common function: $y = [\![x]\!]$

Transformation: Vertical stretch (each y-value is multiplied by 2), then a vertical shift nine units upward



122. $f(x) = 10 - \frac{1}{3}[\![x + 3]\!]$

Horizontal shift of three units to the left, vertical shrink (each y-value is multiplied by $\frac{1}{3}$), reflection in the x-axis and vertical shift of ten units upward of $y = [\![x]\!]$



Section 2.3 Polynomial and Synthetic Division

You should know the following basic techniques and principles of polynomial division.

- The Division Algorithm (Long Division of Polynomials)
- Synthetic Division
- $f(k)$ is equal to the remainder of $f(x)$ divided by $(x - k)$ (the Remainder Theorem).
- $f(k) = 0$ if and only if $(x - k)$ is a factor of $f(x)$.

Vocabulary Check

1. $f(x)$ is the dividend; $d(x)$ is the divisor; $g(x)$ is the quotient; $r(x)$ is the remainder

2. improper; proper **3.** synthetic division **4.** factor **5.** remainder

1. $y_1 = \dfrac{x^2}{x + 2}$ and $y_2 = x - 2 + \dfrac{4}{x + 2}$

$$\begin{array}{r} x - 2 \\ x + 2 \overline{\smash{)}\, x^2 + 0x + 0} \\ \underline{x^2 + 2x} \\ -2x + 0 \\ \underline{-2x - 4} \\ 4 \end{array}$$

Thus, $\dfrac{x^2}{x + 2} = x - 2 + \dfrac{4}{x + 2}$ and $y_1 = y_2$.

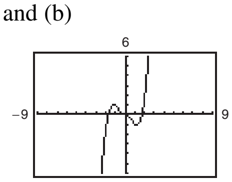
2. $y_1 = \dfrac{x^4 - 3x^2 - 1}{x^2 + 5}$ and $y_2 = x^2 - 8 + \dfrac{39}{x^2 + 5}$

$$\begin{array}{r} x^2 - 8 \\ x^2 + 5 \overline{\smash{)}\, x^4 - 3x^2 - 1} \\ \underline{x^4 + 5x^2} \\ -8x^2 - 1 \\ \underline{-8x^2 - 40} \\ 39 \end{array}$$

Thus, $\dfrac{x^4 - 3x^2 - 1}{x^2 + 5} = x^2 - 8 + \dfrac{39}{x^2 + 5}$ and $y_1 = y_2$.

3. $y_1 = \dfrac{x^5 - 3x^3}{x^2 + 1}$ and $y_2 = x^3 - 4x + \dfrac{4x}{x^2 + 1}$

(a) and (b)

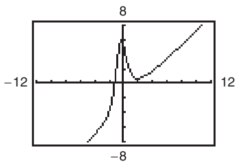


(c)
$$\begin{array}{r} x^3 - 4x \\ x^2 + 0x + 1 \overline{\smash{)}\, x^5 + 0x^4 - 3x^3 + 0x^2 + 0x + 0} \\ \underline{x^5 + 0x^4 + x^3} \qquad\qquad\qquad \\ -4x^3 + 0x^2 + 0x \\ \underline{-4x^3 + 0x^2 - 4x} \\ 4x + 0 \end{array}$$

Thus, $\dfrac{x^5 - 3x^3}{x^2 + 1} = x^3 - 4x + \dfrac{4x}{x^2 + 1}$ and $y_1 = y_2$.

4. $y_1 = \dfrac{x^3 - 2x^2 + 5}{x^2 + x + 1}$ and $y_2 = x - 3 + \dfrac{2(x + 4)}{x^2 + x + 1}$

(a) and (b)



(c)
$$\begin{array}{r}
x - 3 \\
x^2 + x + 1 \overline{)\, x^3 - 2x^2 + 0x + 5} \\
\underline{x^3 + x^2 + x} \\
-3x^2 - x + 5 \\
\underline{-3x^2 - 3x - 3} \\
2x + 8
\end{array}$$

Thus, $\dfrac{x^3 - 2x^2 + 5}{x^2 + x + 1} = x - 3 + \dfrac{2(x + 4)}{x^2 + x + 1}$ and $y_1 = y_2$.

5.
$$\begin{array}{r}
2x + 4 \\
x + 3 \overline{)\, 2x^2 + 10x + 12} \\
\underline{2x^2 + 6x} \\
4x + 12 \\
\underline{4x + 12} \\
0
\end{array}$$

$$\frac{2x^2 + 10x + 12}{x + 3} = 2x + 4$$

6.
$$\begin{array}{r}
5x + 3 \\
x - 4 \overline{)\, 5x^2 - 17x - 12} \\
\underline{5x^2 - 20x} \\
3x - 12 \\
\underline{3x - 12} \\
0
\end{array}$$

$$\frac{5x^2 - 17x - 12}{x - 4} = 5x + 3$$

7.
$$\begin{array}{r}
x^2 - 3x + 1 \\
4x + 5 \overline{)\, 4x^3 - 7x^2 - 11x + 5} \\
\underline{4x^3 + 5x^2} \\
-12x^2 - 11x \\
\underline{-12x^2 - 15x} \\
4x + 5 \\
\underline{4x + 5} \\
0
\end{array}$$

$$\frac{4x^3 - 7x^2 - 11x + 5}{4x + 5} = x^2 - 3x + 1$$

8.
$$\begin{array}{r}
2x^2 - 4x + 3 \\
3x - 2 \overline{)\, 6x^3 - 16x^2 + 17x - 6} \\
\underline{6x^3 - 4x^2} \\
-12x^2 + 17x \\
\underline{-12x^2 + 8x} \\
9x - 6 \\
\underline{9x - 6} \\
0
\end{array}$$

$$\frac{6x^3 - 16x^2 + 17x - 6}{3x - 2} = 2x^2 - 4x + 3$$

9.
$$\begin{array}{r}
x^3 + 3x^2 - 1 \\
x + 2 \overline{)\, x^4 + 5x^3 + 6x^2 - x - 2} \\
\underline{x^4 + 2x^3} \\
3x^3 + 6x^2 \\
\underline{3x^3 + 6x^2} \\
-x - 2 \\
\underline{-x - 2} \\
0
\end{array}$$

$$\frac{x^4 + 5x^3 + 6x^2 - x - 2}{x + 2} = x^3 + 3x^2 - 1$$

10.
$$\begin{array}{r}
x^2 + 7x + 18 \\
x - 3 \overline{)\, x^3 + 4x^2 - 3x - 12} \\
\underline{x^3 - 3x^2} \\
7x^2 - 3x \\
\underline{7x^2 - 21x} \\
18x - 12 \\
\underline{18x - 54} \\
42
\end{array}$$

$$\frac{x^3 + 4x^2 - 3x - 12}{x - 3} = x^2 + 7x + 18 + \frac{42}{x - 3}$$

11.
$$\begin{array}{r}
7\\
x+2\,\overline{)\,7x+3}\\
\underline{7x+14}\\
-11
\end{array}$$

$$\frac{7x+3}{x+2}=7-\frac{11}{x+2}$$

12.
$$\begin{array}{r}
4\\
2x+1\,\overline{)\,8x-5}\\
\underline{8x+4}\\
-9
\end{array}$$

$$\frac{8x-5}{2x+1}=4-\frac{9}{2x+1}$$

13.
$$\begin{array}{r}
3x+5\\
2x^2+0x+1\,\overline{)\,6x^3+10x^2+x+8}\\
\underline{6x^3+0x^2+3x}\\
10x^2-2x+8\\
\underline{10x^2+0x+5}\\
-2x+3
\end{array}$$

$$\frac{6x^3+10x^2+x+8}{2x^2+1}=3x+5-\frac{2x-3}{2x^2+1}$$

14.
$$\begin{array}{r}
x\\
x^2+0x+1\,\overline{)\,x^3+0x^2+0x-9}\\
\underline{x^3+0x^2+x}\\
-x-9
\end{array}$$

$$\frac{x^3-9}{x^2+1}=x-\frac{x+9}{x^2+1}$$

15.
$$\begin{array}{r}
x^2+2x+4\\
x^2-2x+3\,\overline{)\,x^4+0x^3+3x^2+0x+1}\\
\underline{x^4-2x^3+3x^2}\phantom{+0x+1}\\
2x^3+0x^2+0x\phantom{+1}\\
\underline{2x^3-4x^2+6x}\phantom{+1}\\
4x^2-6x+1\\
\underline{4x^2-8x+12}\\
2x-11
\end{array}
\Longrightarrow \frac{x^4+3x^2+1}{x^2-2x+3}=x^2+2x+4+\frac{2x-11}{x^2-2x+3}$$

16.
$$\begin{array}{r}
x^2\\
x^3+0x^2+0x-1\,\overline{)\,x^5+0x^4+0x^3+0x^2+0x+7}\\
\underline{x^5+0x^4+0x^3-x^2}\\
x^2+7
\end{array}$$

$$\frac{x^5+7}{x^3-1}=x^2+\frac{x^2+7}{x^3-1}$$

17.
$$\begin{array}{r}
x+3\\
x^3-3x^2+3x-1\,\overline{)\,x^4+0x^3+0x^2+0x+0}\\
\underline{x^4-3x^3+3x^2-x}\\
3x^3-3x^2+x+0\\
\underline{3x^3-9x^2+9x-3}\\
6x^2-8x+3
\end{array}$$

$$\frac{x^4}{(x-1)^3}=x+3+\frac{6x^2-8x+3}{(x-1)^3}$$

18.
$$\begin{array}{r}
2x\\
x^2-2x+1\,\overline{)\,2x^3-4x^2-15x+5}\\
\underline{2x^3-4x^2+2x}\\
-17x+5
\end{array}$$

$$\frac{2x^3-4x^2-15x+5}{(x-1)^2}=2x-\frac{17x-5}{x^2-2x+1}$$

19.
$$\begin{array}{r|rrrr}
5 & 3 & -17 & 15 & -25\\
 & & 15 & -10 & 25\\
\hline
 & 3 & -2 & 5 & 0
\end{array}$$

$$\frac{3x^3-17x^2+15x-25}{x-5}=3x^2-2x+5$$

20. $$\begin{array}{r|rrrr} -3 & 5 & 18 & 7 & -6 \\ & & -15 & -9 & 6 \\ \hline & 5 & 3 & -2 & 0 \end{array}$$

$$\frac{5x^3 + 18x^2 + 7x - 6}{x + 3} = 5x^2 + 3x - 2$$

21. $$\begin{array}{r|rrrr} -2 & 4 & 8 & -9 & -18 \\ & & -8 & 0 & 18 \\ \hline & 4 & 0 & -9 & 0 \end{array}$$

$$\frac{4x^3 + 8x^2 - 9x - 18}{x + 2} = 4x^2 - 9$$

22. $$\begin{array}{r|rrrr} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$

$$\frac{9x^3 - 18x^2 - 16x + 32}{x - 2} = 9x^2 - 16$$

23. $$\begin{array}{r|rrrr} -10 & -1 & 0 & 75 & -250 \\ & & 10 & -100 & 250 \\ \hline & -1 & 10 & -25 & 0 \end{array}$$

$$\frac{-x^3 + 75x - 250}{x + 10} = -x^2 + 10x - 25$$

24. $$\begin{array}{r|rrrr} 6 & 3 & -16 & 0 & -72 \\ & & 18 & 12 & 72 \\ \hline & 3 & 2 & 12 & 0 \end{array}$$

$$\frac{3x^3 - 16x^2 - 72}{x - 6} = 3x^2 + 2x + 12$$

25. $$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 8 \\ & & 20 & 56 & 224 \\ \hline & 5 & 14 & 56 & 232 \end{array}$$

$$\frac{5x^3 - 6x^2 + 8}{x - 4} = 5x^2 + 14x + 56 + \frac{232}{x - 4}$$

26. $$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

$$\frac{5x^3 + 6x + 8}{x + 2} = 5x^2 - 10x + 26 - \frac{44}{x + 2}$$

27. $$\begin{array}{r|rrrrr} 6 & 10 & -50 & 0 & 0 & -800 \\ & & 60 & 60 & 360 & 2160 \\ \hline & 10 & 10 & 60 & 360 & 1360 \end{array}$$

$$\frac{10x^4 - 50x^3 - 800}{x - 6} = 10x^3 + 10x^2 + 60x + 360 + \frac{1360}{x - 6}$$

28. $$\begin{array}{r|rrrrrr} -3 & 1 & -13 & 0 & 0 & -120 & 80 \\ & & -3 & 48 & -144 & 432 & -936 \\ \hline & 1 & -16 & 48 & -144 & 312 & -856 \end{array}$$

$$\frac{x^5 - 13x^4 - 120x + 80}{x + 3} = x^4 - 16x^3 + 48x^2 - 144x + 312 - \frac{856}{x + 3}$$

29. $$\begin{array}{r|rrrr} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$

$$\frac{x^3 + 512}{x + 8} = x^2 - 8x + 64$$

30. $$\begin{array}{r|rrrr} 9 & 1 & 0 & 0 & -729 \\ & & 9 & 81 & 729 \\ \hline & 1 & 9 & 81 & 0 \end{array}$$

$$\frac{x^3 - 729}{x - 9} = x^2 + 9x + 81$$

31. $$\begin{array}{r|rrrrr} 2 & -3 & 0 & 0 & 0 & 0 \\ & & -6 & -12 & -24 & -48 \\ \hline & -3 & -6 & -12 & -24 & -48 \end{array}$$

$$\frac{-3x^4}{x - 2} = -3x^3 - 6x^2 - 12x - 24 - \frac{48}{x - 2}$$

32. $$\begin{array}{r|rrrrr} -2 & -3 & 0 & 0 & 0 & 0 \\ & & 6 & -12 & 24 & -48 \\ \hline & -3 & 6 & -12 & 24 & -48 \end{array}$$

$$\frac{-3x^4}{x + 2} = -3x^3 + 6x^2 - 12x + 24 - \frac{48}{x + 2}$$

33. $$\begin{array}{r|rrrrr} 6 & -1 & 0 & 0 & 180 & 0 \\ & & -6 & -36 & -216 & -216 \\ \hline & -1 & -6 & -36 & -36 & -216 \end{array}$$

$$\frac{180x - x^4}{x - 6} = -x^3 - 6x^2 - 36x - 36 - \frac{216}{x - 6}$$

34. $$\begin{array}{r|rrrr} -1 & -1 & 2 & -3 & 5 \\ & & 1 & -3 & 6 \\ \hline & -1 & 3 & -6 & 11 \end{array}$$

$$\frac{5 - 3x + 2x^2 - x^3}{x + 1} = -x^2 + 3x - 6 + \frac{11}{x + 1}$$

35.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$

$$\frac{4x^3 + 16x^2 - 23x - 15}{x + (1/2)} = 4x^2 + 14x - 30$$

36.
$$\begin{array}{r|rrrr} \frac{3}{2} & 3 & -4 & 0 & 5 \\ & & \frac{9}{2} & \frac{3}{4} & \frac{9}{8} \\ \hline & 3 & \frac{1}{2} & \frac{3}{4} & \frac{49}{8} \end{array}$$

$$\frac{3x^3 - 4x^2 + 5}{x - (3/2)} = 3x^2 + \frac{1}{2}x + \frac{3}{4} + \frac{49}{8x - 12}$$

37. $f(x) = x^3 - x^2 - 14x + 11,\ k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x - 4)(x^2 + 3x - 2) + 3$

$f(4) = 4^3 - 4^2 - 14(4) + 11 = 3$

38. $f(x) = x^3 - 5x^2 - 11x + 8, k = -2$

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -11 & 8 \\ & & -2 & 14 & -6 \\ \hline & 1 & -7 & 3 & 2 \end{array}$$

$f(x) = (x + 2)(x^2 - 7x + 3) + 2$

$f(-2) = (-2)^3 - 5(-2)^2 - 11(-2) + 8$

$= -8 - 20 + 22 + 8 = 2$

39. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14,\ k = -\frac{2}{3}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$

$f(x) = \left(x + \frac{2}{3}\right)(15x^3 - 6x + 4) + \frac{34}{3}$

$f\left(-\frac{2}{3}\right) = 15\left(-\frac{2}{3}\right)^4 + 10\left(-\frac{2}{3}\right)^3 - 6\left(-\frac{2}{3}\right)^2 + 14 = \frac{34}{3}$

40. $f(x) = 10x^3 - 22x^2 - 3x + 4, k = \frac{1}{5}$

$$\begin{array}{r|rrrr} \frac{1}{5} & 10 & -22 & -3 & 4 \\ & & 2 & -4 & -\frac{7}{5} \\ \hline & 10 & -20 & -7 & \frac{13}{5} \end{array}$$

$f(x) = \left(x - \frac{1}{5}\right)(10x^2 - 20x - 7) + \frac{13}{5}$

$f\left(\frac{1}{5}\right) = 10\left(\frac{1}{5}\right)^3 - 22\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) + 4$

$= \frac{2}{25} - \frac{22}{25} - \frac{3}{5} + 4 = \frac{65}{25} = \frac{13}{5}$

41. $f(x) = x^3 + 3x^2 - 2x - 14,\ k = \sqrt{2}$

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2 + 3\sqrt{2} & 6 \\ \hline & 1 & 3 + \sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$f(x) = \left(x - \sqrt{2}\right)\left[x^2 + \left(3 + \sqrt{2}\right)x + 3\sqrt{2}\right] - 8$

$f\left(\sqrt{2}\right) = \left(\sqrt{2}\right)^3 + 3\left(\sqrt{2}\right)^2 - 2\sqrt{2} - 14 = -8$

42. $f(x) = x^3 + 2x^2 - 5x - 4, k = -\sqrt{5}$

$$\begin{array}{r|rrrr} -\sqrt{5} & 1 & 2 & -5 & -4 \\ & & -\sqrt{5} & -2\sqrt{5} + 5 & 10 \\ \hline & 1 & 2 - \sqrt{5} & -2\sqrt{5} & 6 \end{array}$$

$f(x) = \left(x + \sqrt{5}\right)\left[x^2 + \left(2 - \sqrt{5}\right)x - 2\sqrt{5}\right] + 6$

$f\left(-\sqrt{5}\right) = \left(-\sqrt{5}\right)^3 + 2\left(-\sqrt{5}\right)^2 - 5\left(-\sqrt{5}\right) - 4$

$= -5\sqrt{5} + 10 + 5\sqrt{5} - 4 = 6$

43. $f(x) = -4x^3 + 6x^2 + 12x + 4,\ k = 1 - \sqrt{3}$

$$\begin{array}{r|rrrr} 1 - \sqrt{3} & -4 & 6 & 12 & 4 \\ & & -4 + 4\sqrt{3} & -10 + 2\sqrt{3} & -4 \\ \hline & -4 & 2 + 4\sqrt{3} & 2 + 2\sqrt{3} & 0 \end{array}$$

$f(x) = \left[x - \left(1 - \sqrt{3}\right)\right]\left[-4x^2 + \left(2 + 4\sqrt{3}\right)x + \left(2 + 2\sqrt{3}\right)\right] + 0$

$f\left(1 - \sqrt{3}\right) = -4\left(1 - \sqrt{3}\right)^3 + 6\left(1 - \sqrt{3}\right)^2 + 12\left(1 - \sqrt{3}\right) + 4 = 0$

44. $f(x) = -3x^3 + 8x^2 + 10x - 8, k = 2 + \sqrt{2}$

$$\begin{array}{r|rrrr} 2 + \sqrt{2} & -3 & 8 & 10 & -8 \\ & & -6 - 3\sqrt{2} & -2 - 4\sqrt{2} & 8 \\ \hline & -3 & 2 - 3\sqrt{2} & 8 - 4\sqrt{2} & 0 \end{array}$$

$f(x) = \left(x - 2 - \sqrt{2}\right)\left[-3x^2 + \left(2 - 3\sqrt{2}\right)x + 8 - 4\sqrt{2}\right] + 0$

$f\left(2 + \sqrt{2}\right) = -3\left(2 + \sqrt{2}\right)^3 + 8\left(2 + \sqrt{2}\right)^2 + 10\left(2 + \sqrt{2}\right) - 8$

$= -3\left(20 + 14\sqrt{2}\right) + 8\left(6 + 4\sqrt{2}\right) + 10\left(2 + \sqrt{2}\right) - 8$

$= -60 - 42\sqrt{2} + 48 + 32\sqrt{2} + 20 + 10\sqrt{2} - 8$

$= 0$

45. $f(x) = 4x^3 - 13x + 10$

(a) $$\begin{array}{r|rrrr} 1 & 4 & 0 & -13 & 10 \\ & & 4 & 4 & -9 \\ \hline & 4 & 4 & -9 & 1 \end{array}$$

$f(1) = 1$

(b) $$\begin{array}{r|rrrr} -2 & 4 & 0 & -13 & 10 \\ & & -8 & 16 & -6 \\ \hline & 4 & -8 & 3 & 4 \end{array}$$

$f(-2) = 4$

(c) $$\begin{array}{r|rrrr} \frac{1}{2} & 4 & 0 & -13 & 10 \\ & & 2 & 1 & -6 \\ \hline & 4 & 2 & -12 & 4 \end{array}$$

$f\left(\frac{1}{2}\right) = 4$

(d) $$\begin{array}{r|rrrr} 8 & 4 & 0 & -13 & 10 \\ & & 32 & 256 & 1944 \\ \hline & 4 & 32 & 243 & 1954 \end{array}$$

$f(8) = 1954$

46. $g(x) = x^6 - 4x^4 + 3x^2 + 2$

(a) $$\begin{array}{r|rrrrrrr} 2 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & 2 & 4 & 0 & 0 & 6 & 12 \\ \hline & 1 & 2 & 0 & 0 & 3 & 6 & 14 \end{array}$$

$g(2) = 14$

(b) $$\begin{array}{r|rrrrrrr} -4 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & -4 & 16 & -48 & 192 & -780 & 3120 \\ \hline & 1 & -4 & 12 & -48 & 195 & -780 & 3122 \end{array}$$

$g(-4) = 3122$

(c) $$\begin{array}{r|rrrrrrr} 3 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & 3 & 9 & 15 & 45 & 144 & 432 \\ \hline & 1 & 3 & 5 & 15 & 48 & 144 & 434 \end{array}$$

$g(3) = 434$

(d) $$\begin{array}{r|rrrrrrr} -1 & 1 & 0 & -4 & 0 & 3 & 0 & 2 \\ & & -1 & 1 & 3 & -3 & 0 & 0 \\ \hline & 1 & -1 & -3 & 3 & 0 & 0 & 2 \end{array}$$

$g(-1) = 2$

47. $h(x) = 3x^3 + 5x^2 - 10x + 1$

(a) $$\begin{array}{r|rrrr} 3 & 3 & 5 & -10 & 1 \\ & & 9 & 42 & 96 \\ \hline & 3 & 14 & 32 & 97 \end{array}$$

$h(3) = 97$

(b) $$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 5 & -10 & 1 \\ & & 1 & 2 & -\frac{8}{3} \\ \hline & 3 & 6 & -8 & -\frac{5}{3} \end{array}$$

$h\left(\frac{1}{3}\right) = -\frac{5}{3}$

(c) $$\begin{array}{r|rrrr} -2 & 3 & 5 & -10 & 1 \\ & & -6 & 2 & 16 \\ \hline & 3 & -1 & -8 & 17 \end{array}$$

$h(-2) = 17$

(d) $$\begin{array}{r|rrrr} -5 & 3 & 5 & -10 & 1 \\ & & -15 & 50 & -200 \\ \hline & 3 & -10 & 40 & -199 \end{array}$$

$h(-5) = -199$

48. $f(x) = 0.4x^4 - 1.6x^3 + 0.7x^2 - 2$

(a) $$\begin{array}{r|rrrrr} 1 & 0.4 & -1.6 & 0.7 & 0 & -2 \\ & & 0.4 & -1.2 & -0.5 & -0.5 \\ \hline & 0.4 & -1.2 & -0.5 & -0.5 & -2.5 \end{array}$$

$f(1) = -2.5$

(b) $$\begin{array}{r|rrrrr} -2 & 0.4 & -1.6 & 0.7 & 0 & -2 \\ & & -0.8 & 4.8 & -11 & 22 \\ \hline & 0.4 & -2.4 & 5.5 & -11 & 20 \end{array}$$

$f(-2) = 20$

(c) $$\begin{array}{r|rrrrr} 5 & 0.4 & -1.6 & 0.7 & 0 & -2 \\ & & 2.0 & 2.0 & 13.5 & 67.5 \\ \hline & 0.4 & 0.4 & 2.7 & 13.5 & 65.5 \end{array}$$

$f(5) = 65.5$

(d) $$\begin{array}{r|rrrrr} -10 & 0.4 & -1.6 & 0.7 & 0 & -2 \\ & & -4.0 & 56.0 & -567 & 5670 \\ \hline & 0.4 & -5.6 & 56.7 & -567 & 5668 \end{array}$$

$f(-10) = 5668$

49. $$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$$\begin{aligned} x^3 - 7x + 6 &= (x - 2)(x^2 + 2x - 3) \\ &= (x - 2)(x + 3)(x - 1) \end{aligned}$$

Zeros: $2, -3, 1$

50. $$\begin{array}{r|rrrr} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

$$\begin{aligned} x^3 - 28x - 48 &= (x + 4)(x^2 - 4x - 12) \\ &= (x + 4)(x - 6)(x + 2) \end{aligned}$$

Zeros: $-4, -2, 6$

51.
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$$\begin{aligned} 2x^3 - 15x^2 + 27x - 10 &= \left(x - \tfrac{1}{2}\right)(2x^2 - 14x + 20) \\ &= (2x - 1)(x - 2)(x - 5) \end{aligned}$$

Zeros: $\frac{1}{2}, 2, 5$

52.
$$\begin{array}{r|rrrr} \frac{2}{3} & 48 & -80 & 41 & -6 \\ & & 32 & -32 & 6 \\ \hline & 48 & -48 & 9 & 0 \end{array}$$

$$\begin{aligned} 48x^3 - 80x^2 + 41x - 6 &= \left(x - \tfrac{2}{3}\right)(48x^2 - 48x + 9) \\ &= \left(x - \tfrac{2}{3}\right)(4x - 3)(12x - 3) \\ &= (3x - 2)(4x - 3)(4x - 1) \end{aligned}$$

Zeros: $\frac{2}{3}, \frac{3}{4}, \frac{1}{4}$

53.
$$\begin{array}{r|rrrr} \sqrt{3} & 1 & 2 & -3 & -6 \\ & & \sqrt{3} & 3 + 2\sqrt{3} & 6 \\ \hline & 1 & 2 + \sqrt{3} & 2\sqrt{3} & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{3} & 1 & 2 + \sqrt{3} & 2\sqrt{3} \\ & & -\sqrt{3} & -2\sqrt{3} \\ \hline & 1 & 2 & 0 \end{array}$$

$x^3 + 2x^2 - 3x - 6 = \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x + 2)$

Zeros: $\pm\sqrt{3}, -2$

54.
$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 2 & -2 & -4 \\ & & \sqrt{2} & 2\sqrt{2} + 2 & 4 \\ \hline & 1 & 2 + \sqrt{2} & 2\sqrt{2} & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{2} & 1 & 2 + \sqrt{2} & 2\sqrt{2} \\ & & -\sqrt{2} & -2\sqrt{2} \\ \hline & 1 & 2 & 0 \end{array}$$

$x^3 + 2x^2 - 2x - 4 = \left(x - \sqrt{2}\right)(x + 2)\left(x + \sqrt{2}\right)$

Zeros: $-2, -\sqrt{2}, \sqrt{2}$

55.
$$\begin{array}{r|rrrr} 1 + \sqrt{3} & 1 & -3 & 0 & 2 \\ & & 1 + \sqrt{3} & 1 - \sqrt{3} & -2 \\ \hline & 1 & -2 + \sqrt{3} & 1 - \sqrt{3} & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 - \sqrt{3} & 1 & -2 + \sqrt{3} & 1 - \sqrt{3} \\ & & 1 - \sqrt{3} & -1 + \sqrt{3} \\ \hline & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} x^3 - 3x^2 + 2 &= \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right](x - 1) \\ &= (x - 1)\left(x - 1 - \sqrt{3}\right)\left(x - 1 + \sqrt{3}\right) \end{aligned}$$

Zeros: $1, 1 \pm \sqrt{3}$

56.
$$\begin{array}{r|rrrr} 2 - \sqrt{5} & 1 & -1 & -13 & -3 \\ & & 2 - \sqrt{5} & 7 - 3\sqrt{5} & 3 \\ \hline & 1 & 1 - \sqrt{5} & -6 - 3\sqrt{5} & 0 \end{array}$$

$$\begin{array}{r|rrr} 2 + \sqrt{5} & 1 & 1 - \sqrt{5} & -6 - 3\sqrt{5} \\ & & 2 + \sqrt{5} & 6 + 3\sqrt{5} \\ \hline & 1 & 3 & 0 \end{array}$$

$x^3 - x^2 - 13x - 3 = \left(x - 2 + \sqrt{5}\right)\left(x - 2 - \sqrt{5}\right)(x + 3)$

Zeros: $2 - \sqrt{5}, 2 + \sqrt{5}, -3$

57. $f(x) = 2x^3 + x^2 - 5x + 2$; Factors: $(x + 2), (x - 1)$

(a)
$$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 & 2 & -3 & 1 \\ & & 2 & -1 \\ \hline & 2 & -1 & 0 \end{array}$$

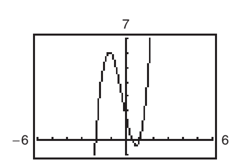
Both are factors of $f(x)$ since the remainders are zero.

(b) The remaining factor of $f(x)$ is $(2x - 1)$.

(c) $f(x) = (2x - 1)(x + 2)(x - 1)$

(d) Zeros: $\frac{1}{2}, -2, 1$

(e)



58. $f(x) = 3x^3 + 2x^2 - 19x + 6$; Factors: $(x + 3)$, $(x - 2)$

(a)
$$\begin{array}{r|rrrr} -3 & 3 & 2 & -19 & 6 \\ & & -9 & 21 & -6 \\ \hline & 3 & -7 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrr} 2 & 3 & -7 & 2 \\ & & 6 & -2 \\ \hline & 3 & -1 & 0 \end{array}$$

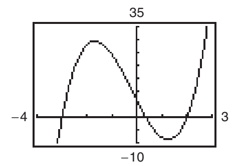
(b) The remaining factor is $(3x - 1)$.

(c) $f(x) = 3x^3 + 2x^2 - 19x + 6$

$= (3x - 1)(x + 3)(x - 2)$

(d) Zeros: $\frac{1}{3}, -3, 2$

(e)



59. $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$; Factors: $(x - 5)$, $(x + 4)$

(a)
$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline & 1 & 1 & -10 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

Both are factors of $f(x)$ since the remainders are zero.

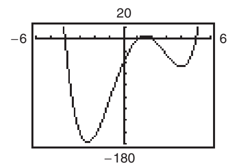
(b) $x^2 - 3x + 2 = (x - 1)(x - 2)$

The remaining factors are $(x - 1)$ and $(x - 2)$.

(c) $f(x) = (x - 1)(x - 2)(x - 5)(x + 4)$

(d) Zeros: $1, 2, 5, -4$

(e)



60. $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$; Factors: $(x + 2)$, $(x - 4)$

(a)
$$\begin{array}{r|rrrrr} -2 & 8 & -14 & -71 & -10 & 24 \\ & & -16 & 60 & 22 & -24 \\ \hline & 8 & -30 & -11 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 8 & -30 & -11 & 12 \\ & & 32 & 8 & -12 \\ \hline & 8 & 2 & -3 & 0 \end{array}$$

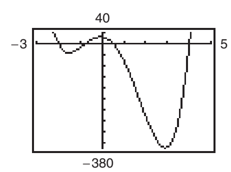
(b) $8x^2 + 2x - 3 = (4x + 3)(2x - 1)$

The remaining factors are $(4x + 3)$ and $(2x - 1)$.

(c) $f(x) = (4x + 3)(2x - 1)(x + 2)(x - 4)$

(d) Zeros: $-\frac{3}{4}, \frac{1}{2}, -2, 4$

(e)



61. $f(x) = 6x^3 + 41x^2 - 9x - 14$; Factors: $(2x + 1)$, $(3x - 2)$

(a)
$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{2}{3} & 6 & 38 & -28 \\ & & 4 & 28 \\ \hline & 6 & 42 & 0 \end{array}$$

Both are factors since the remainders are zero.

(b) $6x + 42 = 6(x + 7)$

This shows that $\dfrac{f(x)}{\left(x + \frac{1}{2}\right)\left(x - \frac{2}{3}\right)} = 6(x + 7)$,

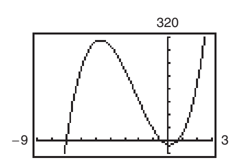
so $\dfrac{f(x)}{(2x + 1)(3x - 2)} = x + 7$.

The remaining factor is $(x + 7)$.

(c) $f(x) = (x + 7)(2x + 1)(3x - 2)$

(d) Zeros: $-7, -\dfrac{1}{2}, \dfrac{2}{3}$

(e)



62. $f(x) = 10x^3 - 11x^2 - 72x + 45$;

Factors: $(2x + 5), (5x - 3)$

(a)
$$\begin{array}{r|rrrr} -\frac{5}{2} & 10 & -11 & -72 & 45 \\ & & -25 & 90 & -45 \\ \hline & 10 & -36 & 18 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{5} & 10 & -36 & 18 \\ & & 6 & -18 \\ \hline & 10 & -30 & 0 \end{array}$$

(b) $10x - 30 = 10(x - 3)$

This shows that $\dfrac{f(x)}{\left(x + \frac{5}{2}\right)\left(x - \frac{3}{5}\right)} = 10(x - 3)$,

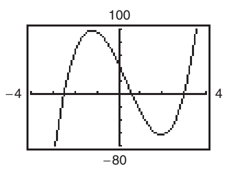
so $\dfrac{f(x)}{(2x + 5)(5x - 3)} = x - 3$.

The remaining factor is $(x - 3)$.

(c) $f(x) = (x - 3)(2x + 5)(5x - 3)$

(d) Zeros: $3, -\dfrac{5}{2}, \dfrac{3}{5}$

(e)



63. $f(x) = 2x^3 - x^2 - 10x + 5$;

Factors: $(2x - 1), \left(x + \sqrt{5}\right)$

(a)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{5} & 2 & 0 & -10 \\ & & -2\sqrt{5} & 10 \\ \hline & 2 & -2\sqrt{5} & 0 \end{array}$$

Both are factors since the remainders are zero.

(b) $2x - 2\sqrt{5} = 2\left(x - \sqrt{5}\right)$

This shows that $\dfrac{f(x)}{\left(x - \frac{1}{2}\right)\left(x + \sqrt{5}\right)} = 2\left(x - \sqrt{5}\right)$,

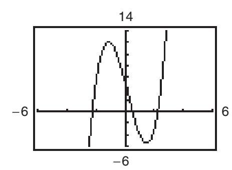
so $\dfrac{f(x)}{(2x - 1)\left(x + \sqrt{5}\right)} = x - \sqrt{5}$.

The remaining factor is $\left(x - \sqrt{5}\right)$.

(c) $f(x) = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)(2x - 1)$

(d) Zeros: $-\sqrt{5}, \sqrt{5}, \dfrac{1}{2}$

(e)



64. $f(x) = x^3 + 3x^2 - 48x - 144$; Factors: $\left(x + 4\sqrt{3}\right), (x + 3)$

(a)
$$\begin{array}{r|rrrr} -3 & 1 & 3 & -48 & -144 \\ & & -3 & 0 & 144 \\ \hline & 1 & 0 & -48 & 0 \end{array}$$

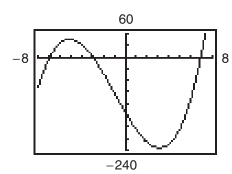
$$\begin{array}{r|rrr} -4\sqrt{3} & 1 & 0 & -48 \\ & & -4\sqrt{3} & 48 \\ \hline & 1 & -4\sqrt{3} & 0 \end{array}$$

(b) The remaining factor is $\left(x - 4\sqrt{3}\right)$.

(c) $f(x) = \left(x - 4\sqrt{3}\right)\left(x + 4\sqrt{3}\right)(x + 3)$

(d) Zeros: $\pm 4\sqrt{3}, -3$

(e)



65. $f(x) = x^3 - 2x^2 - 5x + 10$

(a) The zeros of f are 2 and $\approx \pm 2.236$.

(b) An exact zero is $x = 2$.

(c)
$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 2)(x^2 - 5) \\ &= (x - 2)\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) \end{aligned}$$

66. $g(x) = x^3 - 4x^2 - 2x + 8$

(a) The zeros of g are $x = 4, x \approx -1.414, x \approx 1.414$.

(b) $x = 4$ is an exact zero.

(c)
$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 4)(x^2 - 2) \\ &= (x - 4)\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) \end{aligned}$$

67. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) The zeros of h are $t = -2$, $t \approx 3.732$, $t \approx 0.268$.

(b) An exact zero is $t = -2$.

(c) $\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array}$

$h(t) = (t + 2)(t^2 - 4t + 1)$

By the Quadratic Formula, the zeros of $t^2 - 4t + 1$ are $2 \pm \sqrt{3}$. Thus,

$$h(t) = (t + 2)\left[t - \left(2 + \sqrt{3}\right)\right]\left[t - \left(2 - \sqrt{3}\right)\right]$$
$$= (t + 2)\left(t - 2 - \sqrt{3}\right)\left(t - 2 + \sqrt{3}\right).$$

68. $f(s) = s^3 - 12s^2 + 40s - 24$

(a) The zeros of f are $s = 6$, $s \approx 0.764$, $s \approx 5.236$

(b) $s = 6$ is an exact zero.

(c) $\begin{array}{r|rrrr} 6 & 1 & -12 & 40 & -24 \\ & & 6 & -36 & 24 \\ \hline & 1 & -6 & 4 & 0 \end{array}$

$$f(s) = (s - 6)(s^2 - 6s + 4)$$
$$= (s - 6)\left[s - \left(3 + \sqrt{5}\right)\right]\left[s - \left(3 - \sqrt{5}\right)\right]$$

69. $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3}$

$\begin{array}{r|rrrr} \frac{3}{2} & 4 & -8 & 1 & 3 \\ & & 6 & -3 & -3 \\ \hline & 4 & -2 & -2 & 0 \end{array}$

$$\frac{4x^3 - 8x^2 + x + 3}{x - \frac{3}{2}} = 4x^2 - 2x - 2 = 2(2x^2 - x - 1)$$

Thus, $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3} = 2x^2 - x - 1,\ x \neq \dfrac{3}{2}$.

70. $\dfrac{x^3 + x^2 - 64x - 64}{x + 8}$

$\begin{array}{r|rrrr} -8 & 1 & 1 & -64 & -64 \\ & & -8 & 56 & 64 \\ \hline & 1 & -7 & -8 & 0 \end{array}$

$$\frac{x^3 + x^2 - 64x - 64}{x + 8} = x^2 - 7x - 8,\ x \neq -8$$

71. $\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{x^2 + 3x + 2} = \dfrac{x^4 + 6x^3 + 11x^2 + 6x}{(x + 1)(x + 2)}$

$\begin{array}{r|rrrrr} -1 & 1 & 6 & 11 & 6 & 0 \\ & & -1 & -5 & -6 & 0 \\ \hline & 1 & 5 & 6 & 0 & 0 \end{array}$

$\begin{array}{r|rrrr} -2 & 1 & 5 & 6 & 0 \\ & & -2 & -6 & 0 \\ \hline & 1 & 3 & 0 & 0 \end{array}$

$$\frac{x^4 + 6x^3 + 11x^2 + 6x}{(x + 1)(x + 2)} = x^2 + 3x,\ x \neq -2, -1$$

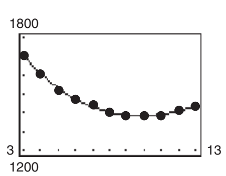
72. $\dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} = \dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{(x + 2)(x - 2)}$

$\begin{array}{r|rrrrr} 2 & 1 & 9 & -5 & -36 & 4 \\ & & 2 & 22 & 34 & -4 \\ \hline & 1 & 11 & 17 & -2 & 0 \end{array}$

$\begin{array}{r|rrrr} -2 & 1 & 11 & 17 & -2 \\ & & -2 & -18 & 2 \\ \hline & 1 & 9 & -1 & 0 \end{array}$

$$\frac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} = x^2 + 9x - 1,\ x \neq \pm 2$$

73. (a) and (b)



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73. —CONTINUED—

(c) $M \approx -0.242t^3 + 12.43t^2 - 173.4t + 2118$

Year, t	Military Personnel	M
3	1705	1703
4	1611	1608
5	1518	1532
6	1472	1473
7	1439	1430
8	1407	1402
9	1386	1388
10	1384	1385
11	1385	1393
12	1412	1409
13	1434	1433

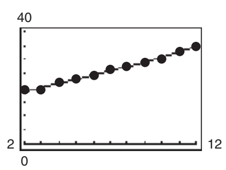
The model is a good fit to the actual data.

(d) $$\begin{array}{r|rrrr} 18 & -0.242 & 12.43 & -173.4 & 2118 \\ & & -4.356 & 145.332 & -505.224 \\ \hline & -0.242 & 8.074 & -28.068 & 1612.776 \end{array}$$

$M(18) \approx 1613$ thousand

No, this model should not be used to predict the number of military personnel in the future. It predicts an increase in military personnel until 2024 and then it decreases and will approach negative infinity quickly.

74. (a) and (b)



(b) $R = 0.0026t^3 - 0.0292t^2 + 1.558t + 15.632$

(c) $$\begin{array}{r|rrrr} 18 & 0.0026 & -0.0292 & 1.558 & 15.632 \\ & & 0.0468 & 0.3168 & 33.7464 \\ \hline & 0.0026 & 0.0176 & 1.8748 & 49.3784 \end{array}$$

For the year 2008, the model predicts a monthly rate of about \$49.38.

75. False. If $(7x + 4)$ is a factor of f, then $-\frac{4}{7}$ is a zero of f.

76. True.

$$\begin{array}{r|rrrrrrr} \frac{1}{2} & 6 & 1 & -92 & 45 & 184 & 4 & -48 \\ & & 3 & 2 & -45 & 0 & 92 & 48 \\ \hline & 6 & 4 & -90 & 0 & 184 & 96 & 0 \end{array}$$

$f(x) = (2x - 1)(x + 1)(x - 2)(x - 3)(3x + 2)(x + 4)$

77. True. The degree of the numerator is greater than the degree of the denominator.

78. $f(x) = (x - k)q(x) + r$

(a) $k = 2,\ r = 5,\ q(x) =$ any quadratic $ax^2 + bx + c$ where $a > 0$. One example:

$f(x) = (x - 2)x^2 + 5 = x^3 - 2x^2 + 5$

(b) $k = -3,\ r = 1,\ q(x) =$ any quadratic $ax^2 + bx + c$ where $a < 0$. One example:

$f(x) = (x + 3)(-x^2) + 1 = -x^3 - 3x^2 + 1$

79.

$$\begin{array}{r} x^{2n} + 6x^n + 9 \\ x^n + 3 \overline{\big) x^{3n} + 9x^{2n} + 27x^n + 27} \\ \underline{x^{3n} + 3x^{2n}} \qquad\qquad\quad \\ 6x^{2n} + 27x^n \qquad \\ \underline{6x^{2n} + 18x^n} \qquad \\ 9x^n + 27 \\ \underline{9x^n + 27} \\ 0 \end{array}$$

$$\frac{x^{3n} + 9x^{2n} + 27x^n + 27}{x^n + 3} = x^{2n} + 6x^n + 9$$

80.

$$\begin{array}{r} x^{2n} - x^n + 3 \\ x^n - 2 \overline{\big) x^{3n} - 3x^{2n} + 5x^n - 6} \\ \underline{x^{3n} - 2x^{2n}} \qquad\qquad\quad \\ -x^{2n} + 5x^n \qquad \\ \underline{-x^{2n} + 2x^n} \qquad \\ 3x^n - 6 \\ \underline{3x^n - 6} \\ 0 \end{array}$$

$$\frac{x^{3n} - 3x^{2n} + 5x^n - 6}{x^n - 2} = x^{2n} - x^n + 3$$

81. A divisor divides evenly into a dividend if the remainder is zero.

82. You can check polynomial division by multiplying the quotient by the divisor. This should yield the original dividend if the multiplication was performed correctly.

83.

$$\begin{array}{r|rrrr} 5 & 1 & 4 & -3 & c \\ & & 5 & 45 & 210 \\ \hline & 1 & 9 & 42 & c + 210 \end{array}$$

To divide evenly, $c + 210$ must equal zero. Thus, c must equal -210.

84.

$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 0 & -2 & 1 & c \\ & & -2 & 4 & -8 & 20 & -42 \\ \hline & 1 & -2 & 4 & -10 & 21 & c - 42 \end{array}$$

To divide evenly, $c - 42$ must equal zero. Thus, c must equal 42.

85. $f(x) = (x + 3)^2(x - 3)(x + 1)^3$

The remainder when $k = -3$ is zero since $(x + 3)$ is a factor of $f(x)$.

86. In this case it is easier to evaluate $f(2)$ directly because $f(x)$ is in factored form. To evaluate using synthetic division you would have to expand each factor and then multiply it all out.

87.

$$9x^2 - 25 = 0$$

$$(3x - 5)(3x + 5) = 0$$

$$3x - 5 = 0 \implies x = \frac{5}{3}$$

$$3x + 5 = 0 \implies x = -\frac{5}{3}$$

88.

$$16x^2 - 21 = 0$$

$$16x^2 = 21$$

$$x^2 = \frac{21}{16}$$

$$x = \pm\sqrt{\frac{21}{16}}$$

$$x = \pm\frac{\sqrt{21}}{4}$$

89.

$$5x^2 - 3x - 14 = 0$$

$$(5x + 7)(x - 2) = 0$$

$$5x + 7 = 0 \implies x = -\tfrac{7}{5}$$

$$x - 2 = 0 \implies x = 2$$

90.

$$8x^2 - 22x + 15 = 0$$

$$(4x - 5)(2x - 3) = 0$$

$$4x - 5 = 0 \quad \text{or} \quad 2x - 3 = 0$$

$$x = \tfrac{5}{4} \quad \text{or} \quad x = \tfrac{3}{2}$$

91. $2x^2 + 6x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(2)(3)}}{2(2)} = \frac{-6 \pm \sqrt{12}}{4}$$

$$= \frac{-3 \pm \sqrt{3}}{2}$$

92. $x^2 + 3x - 3 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-3)}}{2(1)} = \frac{-3 \pm \sqrt{21}}{2}$$

93. $f(x) = (x - 0)(x - 3)(x - 4)$

$= x(x - 3)(x - 4) = x(x^2 - 7x + 12)$

$= x^3 - 7x^2 + 12x$

Note: Any nonzero scalar multiple of $f(x)$ would also have these zeros.

94. $f(x) = (x - (-6))(x - 1)$

$= (x + 6)(x - 1)$

$= x^2 + 5x - 6$

Note: Any nonzero scalar multiple of $f(x)$ would also have these zeros.

95. $f(x) = [x - (-3)]\left[x - \left(1 + \sqrt{2}\right)\right]\left[x - \left(1 - \sqrt{2}\right)\right]$

$= (x + 3)\left[(x - 1) - \sqrt{2}\right]\left[(x - 1) + \sqrt{2}\right]$

$= (x + 3)\left[(x - 1)^2 - \left(\sqrt{2}\right)^2\right]$

$= (x + 3)(x^2 - 2x - 1)$

$= x^3 + x^2 - 7x - 3$

Note: Any nonzero scalar multiple of $f(x)$ would also have these zeros.

96. $f(x) = (x - 1)[x - (-2)]\left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right]$

$= (x - 1)(x + 2)\left[(x - 2) - \sqrt{3}\right]\left[(x - 2) + \sqrt{3}\right]$

$= (x^2 + x - 2)\left[(x - 2)^2 - \left(\sqrt{3}\right)^2\right]$

$= (x^2 + x - 2)(x^2 - 4x + 1)$

$= x^4 - 3x^3 - 5x^2 + 9x - 2$

Note: Any nonzero scalar multiple of $f(x)$ would also have these zeros.

Section 2.4 Complex Numbers

- Standard form: $a + bi$.

  If $b = 0$, then $a + bi$ is a real number.

  If $a = 0$ and $b \neq 0$, then $a + bi$ is a pure imaginary number.
- Equality of Complex Numbers: $a + bi = c + di$ if and only if $a = c$ and $b = d$
- Operations on complex numbers

  (a) Addition: $(a + bi) + (c + di) = (a + c) + (b + d)i$

  (b) Subtraction: $(a + bi) - (c + di) = (a - c) + (b - d)i$

  (c) Multiplication: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

  (d) Division: $\dfrac{a + bi}{c + di} = \dfrac{a + bi}{c + di} \cdot \dfrac{c - di}{c - di} = \dfrac{ac + bd}{c^2 + d^2} + \dfrac{bc - ad}{c^2 + d^2}i$
- The complex conjugate of $a + bi$ is $a - bi$:

  $(a + bi)(a - bi) = a^2 + b^2$
- The additive inverse of $a + bi$ is $-a - bi$.
- $\sqrt{-a} = \sqrt{a}\,i$ for $a > 0$.

Vocabulary Check

1. (a) iii (b) i (c) ii

2. $\sqrt{-1}$; -1

3. principal square

4. complex conjugates

1. $a + bi = -10 + 6i$

$a = -10$

$b = 6$

2. $a + bi = 13 + 4i$

$a = 13$

$b = 4$

3. $(a - 1) + (b + 3)i = 5 + 8i$

$a - 1 = 5 \implies a = 6$

$b + 3 = 8 \implies b = 5$

4. $(a + 6) + 2bi = 6 - 5i$

$2b = -5$

$b = -\frac{5}{2}$

$a + 6 = 6$

$a = 0$

5. $4 + \sqrt{-9} = 4 + 3i$

6. $3 + \sqrt{-16} = 3 + 4i$

7. $2 - \sqrt{-27} = 2 - \sqrt{27}i$

$= 2 - 3\sqrt{3}i$

8. $1 + \sqrt{-8} = 1 + 2\sqrt{2}i$

9. $\sqrt{-75} = \sqrt{75}i = 5\sqrt{3}i$

10. $\sqrt{-4} = 2i$

11. $8 = 8 + 0i = 8$

12. 45

13. $-6i + i^2 = -6i - 1$

$= -1 - 6i$

14. $-4i^2 + 2i = -4(-1) + 2i$

$= 4 + 2i$

15. $\sqrt{-0.09} = \sqrt{0.09}\,i$

$= 0.3i$

16. $\sqrt{-0.0004} = 0.02i$

17. $(5 + i) + (6 - 2i) = 11 - i$

18. $(13 - 2i) + (-5 + 6i) = 8 + 4i$

19. $(8 - i) - (4 - i) = 8 - i - 4 + i$

$= 4$

20. $(3 + 2i) - (6 + 13i) = 3 + 2i - 6 - 13i$

$= -3 - 11i$

21. $\left(-2 + \sqrt{-8}\right) + \left(5 - \sqrt{-50}\right) = -2 + 2\sqrt{2}i + 5 - 5\sqrt{2}i$

$= 3 - 3\sqrt{2}i$

22. $\left(8 + \sqrt{-18}\right) - \left(4 + 3\sqrt{2}i\right) = 8 + 3\sqrt{2}i - 4 - 3\sqrt{2}i$

$= 4$

23. $13i - (14 - 7i) = 13i - 14 + 7i$

$= -14 + 20i$

24. $22 + (-5 + 8i) + 10i = 17 + 18i$

25. $-\left(\frac{3}{2} + \frac{5}{2}i\right) + \left(\frac{5}{3} + \frac{11}{3}i\right) = -\frac{3}{2} - \frac{5}{2}i + \frac{5}{3} + \frac{11}{3}i$

$= -\frac{9}{6} - \frac{15}{6}i + \frac{10}{6} + \frac{22}{6}i$

$= \frac{1}{6} + \frac{7}{6}i$

26. $(1.6 + 3.2i) + (-5.8 + 4.3i) = -4.2 + 7.5i$

27. $(1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2$

$= 3 + i + 2 = 5 + i$

28. $(6 - 2i)(2 - 3i) = 12 - 18i - 4i + 6i^2$

$= 12 - 22i - 6 = 6 - 22i$

29. $6i(5 - 2i) = 30i - 12i^2 = 30i + 12$

$= 12 + 30i$

30. $-8i(9 + 4i) = -72i - 32i^2$

$= 32 - 72i$

31. $\left(\sqrt{14} + \sqrt{10}\,i\right)\left(\sqrt{14} - \sqrt{10}\,i\right) = 14 - 10i^2$

$= 14 + 10 = 24$

32. $\left(\sqrt{3}+\sqrt{15}i\right)\left(\sqrt{3}-\sqrt{15}i\right) = 3 - 15i^2$
$= 3 - 15(-1)$
$= 3 + 15 = 18$

33. $(4+5i)^2 = 16 + 40i + 25i^2$
$= 16 + 40i - 25$
$= -9 + 40i$

34. $(2-3i)^2 = 4 - 12i + 9i^2$
$= 4 - 9 - 12i$
$= -5 - 12i$

35. $(2+3i)^2 + (2-3i)^2 = 4 + 12i + 9i^2 + 4 - 12i + 9i^2$
$= 4 + 12i - 9 + 4 - 12i - 9$
$= -10$

36. $(1-2i)^2 - (1+2i)^2 = 1 - 4i + 4i^2 - (1 + 4i + 4i^2)$
$= 1 - 4i + 4i^2 - 1 - 4i - 4i^2$
$= -8i$

37. The complex conjugate of $6 + 3i$ is $6 - 3i$.
$(6+3i)(6-3i) = 36 - (3i)^2 = 36 + 9 = 45$

38. The complex conjugate of $7 - 12i$ is $7 + 12i$.
$(7-12i)(7+12i) = 49 - 144i^2$
$= 49 - (-144)$
$= 193$

39. The complex conjugate of $-1 - \sqrt{5}i$ is $-1 + \sqrt{5}i$.
$\left(-1-\sqrt{5}i\right)\left(-1+\sqrt{5}i\right) = (-1)^2 - \left(\sqrt{5}i\right)^2$
$= 1 + 5 = 6$

40. The complex conjugate of $-3 + \sqrt{2}i$ is $-3 - \sqrt{2}i$.
$\left(-3+\sqrt{2}i\right)\left(-3-\sqrt{2}i\right) = 9 - 2i^2$
$= 9 - (-2)$
$= 11$

41. The complex conjugate of $\sqrt{-20} = 2\sqrt{5}i$ is $-2\sqrt{5}i$.
$\left(2\sqrt{5}i\right)\left(-2\sqrt{5}i\right) = -20i^2 = 20$

42. The complex conjugate of $\sqrt{-15} = \sqrt{15}i$ is $-\sqrt{15}i$.
$\left(\sqrt{15}i\right)\left(-\sqrt{15}i\right) = -15i^2 = -(-15) = 15$

43. The complex conjugate of $\sqrt{8}$ is $\sqrt{8}$.
$\left(\sqrt{8}\right)\left(\sqrt{8}\right) = 8$

44. The complex conjugate of $1 + \sqrt{8}$ is $1 + \sqrt{8}$.
$\left(1+\sqrt{8}\right)\left(1+\sqrt{8}\right) = 1 + 2\sqrt{8} + 8$
$= 9 + 4\sqrt{2}$

45. $\dfrac{5}{i} = \dfrac{5}{i}\cdot\dfrac{-i}{-i} = \dfrac{-5i}{1} = -5i$

46. $-\dfrac{14}{2i}\cdot\dfrac{-2i}{-2i} = \dfrac{28i}{-4i^2} = \dfrac{28i}{4} = 7i$

47. $\dfrac{2}{4-5i} = \dfrac{2}{4-5i}\cdot\dfrac{4+5i}{4+5i}$
$= \dfrac{2(4+5i)}{16+25} = \dfrac{8+10i}{41} = \dfrac{8}{41} + \dfrac{10}{41}i$

48. $\dfrac{5}{1-i}\cdot\dfrac{1+i}{1+i} = \dfrac{5+5i}{1-i^2} = \dfrac{5+5i}{2} = \dfrac{5}{2} + \dfrac{5}{2}i$

49. $\dfrac{3+i}{3-i} = \dfrac{3+i}{3-i}\cdot\dfrac{3+i}{3+i}$
$= \dfrac{9+6i+i^2}{9+1} = \dfrac{8+6i}{10} = \dfrac{4}{5} + \dfrac{3}{5}i$

50. $\dfrac{6-7i}{1-2i}\cdot\dfrac{1+2i}{1+2i} = \dfrac{6+12i-7i-14i^2}{1-4i^2}$
$= \dfrac{20+5i}{5} = \dfrac{20}{5} + \dfrac{5}{5}i = 4 + i$

51. $\dfrac{6-5i}{i} = \dfrac{6-5i}{i}\cdot\dfrac{-i}{-i}$
$= \dfrac{-6i+5i^2}{1} = -5 - 6i$

52. $\dfrac{8+16i}{2i} \cdot \dfrac{-2i}{-2i} = \dfrac{-16i-32i^2}{-4i^2} = 8-4i$

53. $\dfrac{3i}{(4-5i)^2} = \dfrac{3i}{16-40i+25i^2} = \dfrac{3i}{-9-40i} \cdot \dfrac{-9+40i}{-9+40i}$

$= \dfrac{-27i+120i^2}{81+1600} = \dfrac{-120-27i}{1681}$

$= -\dfrac{120}{1681} - \dfrac{27}{1681}i$

54. $\dfrac{5i}{(2+3i)^2} = \dfrac{5i}{4+12i+9i^2}$

$= \dfrac{5i}{-5+12i} \cdot \dfrac{-5-12i}{-5-12i}$

$= \dfrac{-25i-60i^2}{25-144i^2}$

$= \dfrac{60-25i}{169} = \dfrac{60}{169} - \dfrac{25}{169}i$

55. $\dfrac{2}{1+i} - \dfrac{3}{1-i} = \dfrac{2(1-i)-3(1+i)}{(1+i)(1-i)}$

$= \dfrac{2-2i-3-3i}{1+1}$

$= \dfrac{-1-5i}{2}$

$= -\dfrac{1}{2} - \dfrac{5}{2}i$

56. $\dfrac{2i}{2+i} + \dfrac{5}{2-i} = \dfrac{2i(2-i)}{(2+i)(2-i)} + \dfrac{5(2+i)}{(2+i)(2-i)}$

$= \dfrac{4i-2i^2+10+5i}{4-i^2}$

$= \dfrac{12+9i}{5}$

$= \dfrac{12}{5} + \dfrac{9}{5}i$

57. $\dfrac{i}{3-2i} + \dfrac{2i}{3+8i} = \dfrac{i(3+8i)+2i(3-2i)}{(3-2i)(3+8i)}$

$= \dfrac{3i+8i^2+6i-4i^2}{9+24i-6i-16i^2}$

$= \dfrac{4i^2+9i}{9+18i+16}$

$= \dfrac{-4+9i}{25+18i} \cdot \dfrac{25-18i}{25-18i}$

$= \dfrac{-100+72i+225i-162i^2}{625+324}$

$= \dfrac{-100+297i+162}{949}$

$= \dfrac{62+297i}{949} = \dfrac{62}{949} + \dfrac{297}{949}i$

58. $\dfrac{1+i}{i} - \dfrac{3}{4-i} = \dfrac{(1+i)(4-i)-3i}{i(4-i)}$

$= \dfrac{4-i+4i-i^2-3i}{4i-i^2}$

$= \dfrac{5}{1+4i} \cdot \dfrac{1-4i}{1-4i}$

$= \dfrac{5-20i}{1-16i^2}$

$= \dfrac{5}{17} - \dfrac{20}{17}i$

59. $\sqrt{-6} \cdot \sqrt{-2} = \left(\sqrt{6}i\right)\left(\sqrt{2}i\right) = \sqrt{12}i^2 = \left(2\sqrt{3}\right)(-1)$

$= -2\sqrt{3}$

60. $\sqrt{-5} \cdot \sqrt{-10} = \left(\sqrt{5}i\right)\left(\sqrt{10}i\right)$

$= \sqrt{50}i^2 = 5\sqrt{2}(-1) = -5\sqrt{2}$

61. $\left(\sqrt{-10}\right)^2 = \left(\sqrt{10}i\right)^2 = 10i^2 = -10$

62. $\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$

63. $\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right) = \left(3+\sqrt{5}i\right)\left(7-\sqrt{10}i\right)$

$= 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2$

$= \left(21+\sqrt{50}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i$

$= \left(21+5\sqrt{2}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i$

64. $(2 - \sqrt{-6})^2 = (2 - \sqrt{6}i)(2 - \sqrt{6}i)$

$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6i^2$

$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6(-1)$

$= 4 - 6 - 4\sqrt{6}i$

$= -2 - 4\sqrt{6}i$

65. $x^2 - 2x + 2 = 0;\ a = 1,\ b = -2,\ c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= 1 \pm i$$

66. $x^2 + 6x + 10 = 0;\ a = 1, b = 6, c = 10$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{-4}}{2}$$

$$= -3 \pm i$$

67. $4x^2 + 16x + 17 = 0;\ a = 4,\ b = 16,\ c = 17$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)}$$

$$= \frac{-16 \pm \sqrt{-16}}{8}$$

$$= \frac{-16 \pm 4i}{8} = -2 \pm \frac{1}{2}i$$

68. $9x^2 - 6x + 37 = 0; a = 9, b = -6, c = 37$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)}$$

$$= \frac{6 \pm \sqrt{-1296}}{18}$$

$$= \frac{1}{3} \pm \frac{36i}{18} = \frac{1}{3} \pm 2i$$

69. $4x^2 + 16x + 15 = 0;\ a = 4,\ b = 16,\ c = 15$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)}$$

$$= \frac{-16 \pm \sqrt{16}}{8} = \frac{-16 \pm 4}{8}$$

$$x = -\frac{12}{8} = -\frac{3}{2} \quad \text{or} \quad x = -\frac{20}{8} = -\frac{5}{2}$$

70. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}$$

$$= \frac{4 \pm \sqrt{-176}}{32}$$

$$= \frac{4 \pm 4\sqrt{11}\,i}{32}$$

$$= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

71. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{-72}}{6}$$

$$= \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i$$

72. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$

$14x^2 - 12x + 5 = 0; a = 14, b = -12, c = 5$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)}$$

$$= \frac{12 \pm \sqrt{-136}}{28}$$

$$= \frac{12 \pm 2i\sqrt{34}}{28}$$

$$= \frac{3}{7} \pm \frac{\sqrt{34}}{14}i$$

73. $1.4x^2 - 2x - 10 = 0$ Multiply both sides by 5.

$7x^2 - 10x - 50 = 0$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)}$$

$$= \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14}$$

$$= \frac{5 \pm 5\sqrt{15}}{7} = \frac{5}{7} \pm \frac{5\sqrt{15}}{7}$$

74. $4.5x^2 - 3x + 12 = 0; a = 4.5, b = -3, c = 12$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)}$$

$$= \frac{3 \pm \sqrt{-207}}{9} = \frac{3 \pm 3i\sqrt{23}}{9} = \frac{1}{3} \pm \frac{\sqrt{23}}{3}i$$

75. $-6i^3 + i^2 = -6i^2 i + i^2$

$$= -6(-1)i + (-1)$$

$$= 6i - 1$$

$$= -1 + 6i$$

76. $4i^2 - 2i^3 = -4 + 2i$

77. $-5i^5 = -5i^2 i^2 i$

$$= -5(-1)(-1)i = -5i$$

78. $(-i)^3 = (-1)(i^3) = (-1)(-i) = i$

79. $\left(\sqrt{-75}\right)^3 = \left(5\sqrt{3}i\right)^3$

$$= 5^3\left(\sqrt{3}\right)^3 i^3$$

$$= 125\left(3\sqrt{3}\right)(-1)i$$

$$= -375\sqrt{3}i$$

80. $\left(\sqrt{-2}\right)^6 = \left(\sqrt{2}i\right)^6 = 8i^6 = 8i^4 i^2 = -8$

81. $\dfrac{1}{i^3} = \dfrac{1}{-i} = \dfrac{1}{-i} \cdot \dfrac{i}{i} = \dfrac{i}{-i^2} = \dfrac{i}{1} = i$

82. $\dfrac{1}{(2i)^3} = \dfrac{1}{8i^3} = \dfrac{1}{-8i} \cdot \dfrac{8i}{8i} = \dfrac{8i}{-64i^2} = \dfrac{1}{8}i$

83. (a) $z_1 = 9 + 16i, z_2 = 20 - 10i$

(b) $\dfrac{1}{z} = \dfrac{1}{z_1} + \dfrac{1}{z_2} = \dfrac{1}{9 + 16i} + \dfrac{1}{20 - 10i} = \dfrac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \dfrac{29 + 6i}{340 + 230i}$

$$z = \left(\frac{340 + 230i}{29 + 6i}\right)\left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11{,}240 + 4630i}{877} = \frac{11{,}240}{877} + \frac{4630}{877}i$$

84. (a) $(2)^3 = 8$

(b) $\left(-1 + \sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3$

$$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3$$

$$= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i$$

$$= 8$$

(c) $\left(-1 - \sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3$

$$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3$$

$$= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i$$

$$= 8$$

85. (a) $2^4 = 16$

(b) $(-2)^4 = 16$

(c) $(2i)^4 = 2^4 i^4 = 16(1) = 16$

(d) $(-2i)^4 = (-2)^4 i^4 = 16(1) = 16$

86. (a) $i^{40} = (i^4)^{10} = (1)^{10} = 1$

(b) $i^{25} = (i^4)^6 \cdot i = (1)^6 i = i$

(c) $i^{50} = (i^4)^{12}(i^2) = (1)(-1) = -1$

(d) $i^{67} = (i^4)^{16}(i^3) = (1)(-i) = -i$

87. False, if $b = 0$ then $a + bi = a - bi = a$.

That is, if the complex number is real, the number equals its conjugate.

88. True

$$x^4 - x^2 + 14 = 56$$

$$\left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 \stackrel{?}{=} 56$$

$$36 + 6 + 14 \stackrel{?}{=} 56$$

$$56 = 56$$

89. False

$$\begin{aligned} i^{44} + i^{150} - i^{74} - i^{109} + i^{61} &= (i^4)^{11} + (i^4)^{37}(i^2) - (i^4)^{18}(i^2) - (i^4)^{27}(i) + (i^4)^{15}(i) \\ &= (1)^{11} + (1)^{37}(-1) - (1)^{18}(-1) - (1)^{27}(i) + (1)^{15}(i) \\ &= 1 + (-1) + 1 - i + i = 1 \end{aligned}$$

90. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$

91. $$\begin{aligned} (a_1 + b_1 i)(a_2 + b_2 i) &= a_1a_2 + a_1b_2 i + a_2b_1 i + b_1b_2 i^2 \\ &= (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i \end{aligned}$$

The complex conjugate of this product is $(a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i$.

The product of the complex conjugates is:

$$\begin{aligned} (a_1 - b_1 i)(a_2 - b_2 i) &= a_1a_2 - a_1b_2 i - a_2b_1 i + b_1b_2 i^2 \\ &= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i \end{aligned}$$

Thus, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

92. $(a_1 + b_1 i) + (a_2 + b_2 i) = (a_1 + a_2) + (b_1 + b_2)i$

The complex conjugate of this sum is $(a_1 + a_2) - (b_1 + b_2)i$.

The sum of the complex conjugates is $(a_1 - b_1 i) + (a_2 - b_2 i) = (a_1 + a_2) - (b_1 + b_2)i$.

Thus, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

93. $(4 + 3x) + (8 - 6x - x^2) = -x^2 - 3x + 12$

94. $$\begin{aligned} (x^3 - 3x^2) - (6 - 2x - 4x^2) &= x^3 - 3x^2 - 6 + 2x + 4x^2 \\ &= x^3 + x^2 + 2x - 6 \end{aligned}$$

95. $$\begin{aligned} \left(3x - \tfrac{1}{2}\right)(x + 4) &= 3x^2 + 12x - \tfrac{1}{2}x - 2 \\ &= 3x^2 + \tfrac{23}{2}x - 2 \end{aligned}$$

96. $$\begin{aligned} (2x - 5)^2 &= (2x)^2 - 2(2x)(5) + (5)^2 \\ &= 4x^2 - 20x + 25 \end{aligned}$$

97. $$\begin{aligned} -x - 12 &= 19 \\ -x &= 31 \\ x &= -31 \end{aligned}$$

98. $$\begin{aligned} 8 - 3x &= -34 \\ -3x &= -42 \\ x &= 14 \end{aligned}$$

99. $$\begin{aligned} 4(5x - 6) - 3(6x + 1) &= 0 \\ 20x - 24 - 18x - 3 &= 0 \\ 2x - 27 &= 0 \\ 2x &= 27 \\ x &= \frac{27}{2} \end{aligned}$$

100. $$\begin{aligned} 5[x - (3x + 11)] &= 20x - 15 \\ 5x - 15x - 55 &= 20x - 15 \\ -30x &= 40 \\ x &= \frac{40}{-30} = -\frac{4}{3} \end{aligned}$$

101. $$\begin{aligned} V &= \frac{4}{3}\pi a^2 b \\ 3V &= 4\pi a^2 b \\ \frac{3V}{4\pi b} &= a^2 \\ \sqrt{\frac{3V}{4\pi b}} &= a \\ a &= \frac{1}{2}\sqrt{\frac{3V}{\pi b}} = \frac{\sqrt{3V\pi b}}{2\pi b} \end{aligned}$$

102. $$\begin{aligned} F &= \alpha\frac{m_1m_2}{r^2} \\ r^2 &= \alpha\frac{m_1m_2}{F} \\ r &= \sqrt{\frac{\alpha m_1m_2}{F}} = \frac{\sqrt{\alpha m_1m_2}}{\sqrt{F}} \cdot \frac{\sqrt{F}}{\sqrt{F}} = \frac{\sqrt{\alpha m_1m_2F}}{F} \end{aligned}$$

103. Let x = # liters withdrawn and replaced.

$$\begin{aligned} 0.50(5 - x) + 1.00x &= 0.60(5) \\ 2.50 - 0.50x + 1.00x &= 3.00 \\ 0.50x &= 0.50 \\ x &= 1 \text{ liter} \end{aligned}$$

Section 2.5 Zeros of Polynomial Functions

- You should know that if f is a polynomial of degree $n > 0$, then f has at least one zero in the complex number system.
- You should know the Linear Factorization Theorem.
- You should know the Rational Zero Test.
- You should know shortcuts for the Rational Zero Test. Possible rational zeros $= \dfrac{\text{factors of constant term}}{\text{factors of leading coefficient}}$
  (a) Use a graphing or programmable calculator.
  (b) Sketch a graph.
  (c) After finding a root, use synthetic division to reduce the degree of the polynomial.
- You should know that if $a + bi$ is a complex zero of a polynomial f, with real coefficients, then $a - bi$ is also a complex zero of f.
- You should know the difference between a factor that is irreducible over the rationals (such as $x^2 - 7$) and a factor that is irreducible over the reals (such as $x^2 + 9$).
- You should know Descartes's Rule of Signs. (For a polynomial with real coefficients and a non-zero constant term.)
  (a) The number of positive real zeros of f is either equal to the number of variations of sign of f or is less than that number by an even integer.
  (b) The number of negative real zeros of f is either equal to the number of variations in sign of $f(-x)$ or is less than that number by an even integer.
  (c) When there is only one variation in sign, there is exactly one positive (or negative) real zero.
- You should be able to observe the last row obtained from synthetic division in order to determine upper or lower bounds.
  (a) If the test value is positive and all of the entries in the last row are positive or zero, then the test value is an upper bound.
  (b) If the test value is negative and the entries in the last row alternate from positive to negative, then the test value is a lower bound. (Zero entries count as positive or negative.)

Vocabulary Check

1. Fundamental Theorem of Algebra
2. Linear Factorization Theorem
3. Rational Zero
4. conjugate
5. irreducible; reals
6. Descarte's Rule of Signs
7. lower; upper

1. $f(x) = x(x - 6)^2$

The zeros are: $x = 0, x = 6$

2. $f(x) = x^2(x + 3)(x^2 - 1) = x^2(x + 3)(x + 1)(x - 1)$

The five zeros are: $0, 0, -3, \pm 1$

3. $g(x) = (x - 2)(x + 4)^3$

The zeros are: $x = 2, x = -4$

4. $f(x) = (x + 5)(x - 8)^2$

The three zeros are: $-5, 8, 8$

5. $f(x) = (x + 6)(x + i)(x - i)$

The three zeros are:

$x = -6, x = -i, x = i$

6. $h(t) = (t - 3)(t - 2)(t - 3i)(t + 3i)$

The four zeros are: $3, 2, \pm 3i$

7. $f(x) = x^3 + 3x^2 - x - 3$

Possible rational zeros: $\pm 1, \pm 3$

Zeros shown on graph: $-3, -1, 1$

8. $f(x) = x^3 - 4x^2 - 4x + 16$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16$

Zeros shown on graph: $-2, 2, 4$

9. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

Possible rational zeros: $\pm1, \pm3, \pm5, \pm9, \pm15, \pm45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Zeros shown on graph: $-1, \frac{3}{2}, 3, 5$

10. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

Possible rational zeros: $\pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}$

Zeros shown on graph: $-1, -\frac{1}{2}, \frac{1}{2}, 1, 2$

11. $f(x) = x^3 - 6x^2 + 11x - 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$$\begin{aligned} x^3 - 6x^2 + 11x - 6 &= (x - 1)(x^2 - 5x + 6) \\ &= (x - 1)(x - 2)(x - 3) \end{aligned}$$

Thus, the rational zeros are 1, 2, and 3.

12. $f(x) = x^3 - 7x - 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$f(x) = (x - 3)(x^2 + 3x + 2) = (x - 3)(x + 2)(x + 1)$

Thus, the rational zeros are $-2, -1, 3$.

13. $g(x) = x^3 - 4x^2 - x + 4 = x^2(x - 4) - 1(x - 4)$

$$\begin{aligned} &= (x - 4)(x^2 - 1) \\ &= (x - 4)(x - 1)(x + 1) \end{aligned}$$

Thus, the rational zeros of $g(x)$ are 4 and ±1.

14. $h(x) = x^3 - 9x^2 + 20x - 12$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 20 & -12 \\ & & 1 & -8 & 12 \\ \hline & 1 & -8 & 12 & 0 \end{array}$$

$$\begin{aligned} h(x) &= (x - 1)(x^2 - 8x + 12) \\ &= (x - 1)(x - 2)(x - 6) \end{aligned}$$

Thus, the rational zeros are 1, 2, 6.

15. $h(t) = t^3 + 12t^2 + 21t + 10$

Possible rational zeros: $\pm1, \pm2, \pm5, \pm10$

$$\begin{array}{r|rrrr} -1 & 1 & 12 & 21 & 10 \\ & & -1 & -11 & -10 \\ \hline & 1 & 11 & 10 & 0 \end{array}$$

$$\begin{aligned} t^3 + 12t^2 + 21t + 10 &= (t + 1)(t^2 + 11t + 10) \\ &= (t + 1)(t + 1)(t + 10) \\ &= (t + 1)^2(t + 10) \end{aligned}$$

Thus, the rational zeros are -1 and -10.

16. $p(x) = x^3 - 9x^2 + 27x - 27$

Possible rational zeros: $\pm1, \pm3, \pm9, \pm27$

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 27 & -27 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 3)(x^2 - 6x + 9) \\ &= (x - 3)(x - 3)(x - 3) \end{aligned}$$

Thus, the rational zero is 3.

17. $C(x) = 2x^3 + 3x^2 - 1$

Possible rational zeros: $\pm1, \pm\frac{1}{2}$

$$\begin{array}{r|rrrr} -1 & 2 & 3 & 0 & -1 \\ & & -2 & -1 & 1 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} 2x^3 + 3x^2 - 1 &= (x + 1)(2x^2 + x - 1) \\ &= (x + 1)(x + 1)(2x - 1) \\ &= (x + 1)^2(2x - 1) \end{aligned}$$

Thus, the rational zeros are -1 and $\frac{1}{2}$.

18. $f(x) = 3x^3 - 19x^2 + 33x - 9$

Possible rational zeros: $\pm 1, \pm 3, \pm 9, \pm\frac{1}{3}$

$$\begin{array}{r|rrrr} 3 & 3 & -19 & 33 & -9 \\ & & 9 & -30 & 9 \\ \hline & 3 & -10 & 3 & 0 \end{array}$$

$f(x) = (x - 3)(3x^2 - 10x + 3) = (x - 3)(3x - 1)(x - 3)$

Thus, the rational zeros are $3, \frac{1}{3}$.

19. $f(x) = 9x^4 - 9x^3 - 58x^2 + 4x + 24$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}, \pm\frac{8}{9}$

$$\begin{array}{r|rrrrr} -2 & 9 & -9 & -58 & 4 & 24 \\ & & -18 & 54 & 8 & -24 \\ \hline & 9 & -27 & -4 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 9 & -27 & -4 & 12 \\ & & 27 & 0 & -12 \\ \hline & 9 & 0 & -4 & 0 \end{array}$$

$$\begin{aligned} 9x^4 - 9x^3 - 58x^2 + 4x - 24 &= (x + 2)(x - 3)(9x^2 - 4) \\ &= (x + 2)(x - 3)(3x - 2)(3x + 2) \end{aligned}$$

Thus, the rational zeros are $-2, 3$, and $\pm\frac{2}{3}$.

20. $f(x) = 2x^4 - 15x^3 + 23x^2 + 15x - 25$

Possible rational zeros: $\pm 1, \pm 5, \pm 25, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{25}{2}$

$$\begin{array}{r|rrrrr} 5 & 2 & -15 & 23 & 15 & -25 \\ & & 10 & -25 & -10 & 25 \\ \hline & 2 & -5 & -2 & 5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & -5 & -2 & 5 \\ & & 2 & -3 & -5 \\ \hline & 2 & -3 & -5 & 0 \end{array}$$

$$\begin{array}{r|rrr} -1 & 2 & -3 & -5 \\ & & -2 & 5 \\ \hline & 2 & -5 & 0 \end{array}$$

$f(x) = (x - 5)(x - 1)(x + 1)(2x - 5)$

Thus, the rational zeros are $5, 1, -1, \frac{5}{2}$.

21. $z^4 - z^3 - 2z - 4 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$z^4 - z^3 - 2z - 4 = (z + 1)(z - 2)(z^2 + 2)$

The only real zeros are -1 and 2.

22. $x^4 - 13x^2 - 12x = 0$

$x(x^3 - 13x - 12) = 0$

Possible rational zeros of $x^3 - 13x - 12$:
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$x(x + 1)(x^2 - x - 12) = 0$

$x(x + 1)(x - 4)(x + 3) = 0$

The real zeros are $0, -1, 4, -3$.

23. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

$$\begin{array}{r|rrrrr} 1 & 2 & 7 & -26 & 23 & -6 \\ & & 2 & 9 & -17 & 6 \\ \hline & 2 & 9 & -17 & 6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -6 & 2 & 9 & -17 & 6 \\ & & -12 & 18 & -6 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$2y^4 + 7y^3 - 26y^2 + 23y - 6 = (y - 1)(y + 6)(2y^2 - 3y + 1) = (y - 1)(y + 6)(2y - 1)(y - 1) = (y - 1)^2(y + 6)(2y - 1)$

The only real zeros are $1, -6$, and $\frac{1}{2}$.

24. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

$x(x^4 - x^3 - 3x^2 + 5x - 2) = 0$

Possible rational zeros of $x^4 - x^3 - 3x^2 + 5x - 2$: $\pm1, \pm2$

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -3 & 5 & -2 \\ & & 1 & 0 & -3 & 2 \\ \hline & 1 & 0 & -3 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$x(x - 1)(x + 2)(x^2 - 2x + 1) = 0$

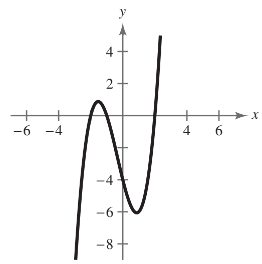
$x(x - 1)(x + 2)(x - 1)(x - 1) = 0$

The real zeros are $-2, 0, 1$.

25. $f(x) = x^3 + x^2 - 4x - 4$

(a) Possible rational zeros: $\pm1, \pm2, \pm4$

(b)

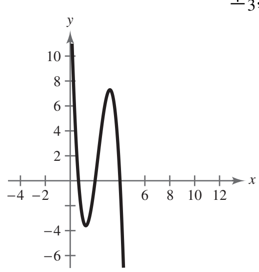


(c) The zeros are: $-2, -1, 2$

26. $f(x) = -3x^3 + 20x^2 - 36x + 16$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$

(b)

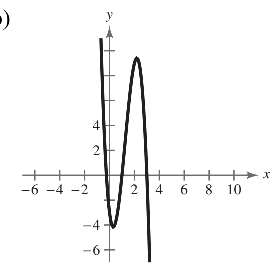


(c) Real zeros: $\frac{2}{3}, 2, 4$

27. $f(x) = -4x^3 + 15x^2 - 8x - 3$

(a) Possible rational zeros: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

(b)

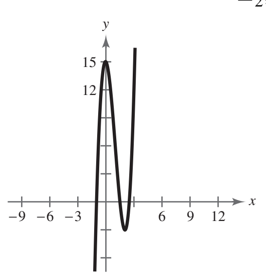


(c) The zeros are: $-\frac{1}{4}, 1, 3$

28. $f(x) = 4x^3 - 12x^2 - x + 15$

(a) Possible rational zeros: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}$

(b)

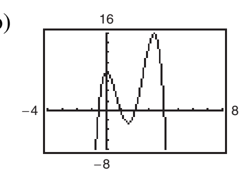


(c) Real zeros: $-1, \frac{3}{2}, \frac{5}{2}$

29. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{2}$

(b)

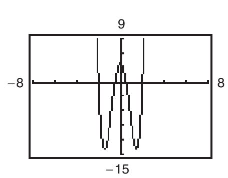


(c) The zeros are: $-\frac{1}{2}, 1, 2, 4$

30. $f(x) = 4x^4 - 17x^2 + 4$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm\frac{1}{2}, \pm\frac{1}{4}$

(b)

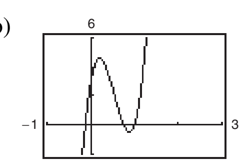


(c) Real zeros: $\pm2, \pm\frac{1}{2}$

31. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) Possible rational zeros: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{1}{32}, \pm\frac{3}{32}$

(b)

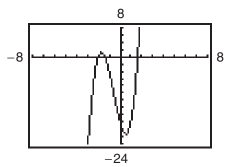


(c) The zeros are: $-\frac{1}{8}, \frac{3}{4}, 1$

32. $f(x) = 4x^3 + 7x^2 - 11x - 18$

(a) Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$

(b)



(c) Real zeros: $-2, \dfrac{1}{8} \pm \dfrac{\sqrt{145}}{8}$

33. $f(x) = x^4 - 3x^2 + 2$

(a) From the calculator we have $x = \pm 1$ and $x \approx \pm 1.414$.

(b) An exact zero is $x = 1$.

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & 2 \\ & & 1 & 1 & -2 & -2 \\ \hline & 1 & 1 & -2 & -2 & 0 \end{array}$$

(c)
$$\begin{array}{r|rrrr} -1 & 1 & 1 & -2 & -2 \\ & & -1 & 0 & 2 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 1)(x + 1)(x^2 - 2) \\ &= (x - 1)(x + 1)\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) \end{aligned}$$

34. $P(t) = t^4 - 7t^2 + 12$

(a) $t = \pm 2, \pm 1.732$

(b)
$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -7 & 0 & 12 \\ & & 2 & 4 & -6 & -12 \\ \hline & 1 & 2 & -3 & -6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -3 & -6 \\ & & -2 & 0 & 6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

(c)
$$\begin{aligned} P(t) &= (t - 2)(t + 2)(t^2 - 3) \\ &= (t - 2)(t + 2)\left(t - \sqrt{3}\right)\left(t + \sqrt{3}\right) \end{aligned}$$

35. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $h(x) = x(x^4 - 7x^3 + 10x^2 + 14x - 24)$

From the calculator we have $x = 0, 3, 4$ and $x \approx \pm 1.414$.

(b) An exact zero is $x = 3$.

$$\begin{array}{r|rrrrr} 3 & 1 & -7 & 10 & 14 & -24 \\ & & 3 & -12 & -6 & 24 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

(c)
$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{aligned} h(x) &= x(x - 3)(x - 4)(x^2 - 2) \\ &= x(x - 3)(x - 4)\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) \end{aligned}$$

36. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) $x = \pm 3, 1.5, 0.333$

(b)
$$\begin{array}{r|rrrrr} 3 & 6 & -11 & -51 & 99 & -27 \\ & & 18 & 21 & -90 & 27 \\ \hline & 6 & 7 & -30 & 9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 6 & 7 & -30 & 9 \\ & & -18 & 33 & -9 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

(c)
$$\begin{aligned} g(x) &= (x - 3)(x + 3)(6x^2 - 11x + 3) \\ &= (x - 3)(x + 3)(3x - 1)(2x - 3) \end{aligned}$$

37.
$$\begin{aligned} f(x) &= (x - 1)(x - 5i)(x + 5i) \\ &= (x - 1)(x^2 + 25) \\ &= x^3 - x^2 + 25x - 25 \end{aligned}$$

Note: $f(x) = a(x^3 - x^2 + 25x - 25)$, where a is any nonzero real number, has the zeros 1 and $\pm 5i$.

38.
$$\begin{aligned} f(x) &= (x - 4)(x - 3i)(x + 3i) \\ &= (x - 4)(x^2 + 9) \\ &= x^3 - 4x^2 + 9x - 36 \end{aligned}$$

Note: $f(x) = a(x^3 - 4x^2 + 9x - 36)$, where a is any real number, has the zeros 4, $3i$ and $-3i$.

39.
$$\begin{aligned} f(x) &= (x - 6)[x - (-5 + 2i)][x - (-5 - 2i)] \\ &= (x - 6)[(x + 5) - 2i][(x + 5) + 2i] \\ &= (x - 6)[(x + 5)^2 - (2i)^2] \\ &= (x - 6)(x^2 + 10x + 25 + 4) \\ &= (x - 6)(x^2 + 10x + 29) \\ &= x^3 + 4x^2 - 31x - 174 \end{aligned}$$

Note: $f(x) = a(x^3 + 4x^2 - 31x - 174)$, where a is any nonzero real number, has the zeros 6, and $-5 \pm 2i$.

40. $f(x) = (x - 2)(x - 4 - i)(x - 4 + i)$

$= (x - 2)(x^2 - 8x + 17)$

$= x^3 - 10x^2 + 33x - 34$

Note: $f(x) = a(x^3 - 10x^2 + 33x - 34)$ where a is any real number, has the zeros $2, 4 \pm i$.

41. If $3 + \sqrt{2}i$ is a zero, so is its conjugate, $3 - \sqrt{2}i$.

$f(x) = (3x - 2)(x + 1)\left[x - \left(3 + \sqrt{2}i\right)\right]\left[x - \left(3 - \sqrt{2}i\right)\right]$

$= (3x - 2)(x + 1)\left[(x - 3) - \sqrt{2}i\right]\left[(x - 3) + \sqrt{2}i\right]$

$= (3x^2 + x - 2)\left[(x - 3)^2 - \left(\sqrt{2}i\right)^2\right]$

$= (3x^2 + x - 2)(x^2 - 6x + 9 + 2)$

$= (3x^2 + x - 2)(x^2 - 6x + 11)$

$= 3x^4 - 17x^3 + 25x^2 + 23x - 22$

Note: $f(x) = a(3x^4 - 17x^3 + 25x^2 + 23x - 22)$, where a is any nonzero real number, has the zeros $\frac{2}{3}$, -1, and $3 \pm \sqrt{2}i$.

42. If $1 + \sqrt{3}i$ is a zero, so is its conjugate, $1 - \sqrt{3}i$.

$f(x) = (x + 5)^2\left(x - 1 - \sqrt{3}i\right)\left(x - 1 + \sqrt{3}i\right)$

$= (x^2 + 10x + 25)(x^2 - 2x + 4)$

$= x^4 + 8x^3 + 9x^2 - 10x + 100$

Note: $f(x) = a(x^4 + 8x^3 + 9x^2 - 10x + 100)$, where a is any real number, has the zeros $-5, -5, 1 \pm \sqrt{3}i$.

43. $f(x) = x^4 + 6x^2 - 27$

(a) $f(x) = (x^2 + 9)(x^2 - 3)$

(b) $f(x) = (x^2 + 9)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

(c) $f(x) = (x + 3i)(x - 3i)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

44. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$

$$
\begin{array}{r}
x^2 - 2x + 3 \\
x - 6 \,\big)\overline{x^4 - 2x^3 - 3x^2 + 12x - 18} \\
\underline{x^4 \qquad\quad - 6x^2 \qquad\qquad} \\
-2x^3 + 3x^2 + 12x \qquad \\
\underline{-2x^3 \qquad\quad + 12x \qquad} \\
3x^2 \qquad - 18 \\
\underline{3x^2 \qquad - 18} \\
0
\end{array}
$$

(a) $f(x) = (x^2 - 6)(x^2 - 2x + 3)$

(b) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)(x^2 - 2x + 3)$

(c) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x - 1 - \sqrt{2}i\right)\left(x - 1 + \sqrt{2}i\right)$

45. $f(x) = x^4 - 4x^3 + 5x^2 - 2x - 6$

$$
\begin{array}{r}
x^2 - 2x + 3 \\
x^2 - 2x - 2 \,\big)\overline{x^4 - 4x^3 + 5x^2 - 2x - 6} \\
\underline{x^4 - 2x^3 - 2x^2 \qquad\qquad} \\
-2x^3 + 7x^2 - 2x \\
\underline{-2x^3 + 4x^2 + 4x} \\
3x^2 - 6x - 6 \\
\underline{3x^2 - 6x - 6} \\
0
\end{array}
$$

$f(x) = (x^2 - 2x - 2)(x^2 - 2x + 3)$

(a) $f(x) = (x^2 - 2x - 2)(x^2 - 2x + 3)$

(b) $f(x) = \left(x - 1 + \sqrt{3}\right)\left(x - 1 - \sqrt{3}\right)(x^2 - 2x + 3)$

(c) $f(x) = \left(x - 1 + \sqrt{3}\right)\left(x - 1 - \sqrt{3}\right)\left(x - 1 + \sqrt{2}\,i\right)\left(x - 1 - \sqrt{2}\,i\right)$

Note: Use the Quadratic Formula for (b) and (c).

46. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$

$$
\begin{array}{r}
x^2 - 3x - 5 \\
x^2 + 4 \,\big)\overline{x^4 - 3x^3 - x^2 - 12x - 20} \\
\underline{x^4 \qquad\quad + 4x^2 \qquad\qquad\qquad} \\
-3x^3 - 5x^2 - 12x \qquad \\
\underline{-3x^3 \qquad\quad - 12x \qquad} \\
-5x^2 \qquad -20 \\
\underline{-5x^2 \qquad -20} \\
0
\end{array}
$$

(a) $f(x) = (x^2 + 4)(x^2 - 3x - 5)$

(b) $f(x) = (x^2 + 4)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

(c) $f(x) = (x + 2i)(x - 2i)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

47. $f(x) = 2x^3 + 3x^2 + 50x + 75$

Since $5i$ is a zero, so is $-5i$.

$$\begin{array}{r|rrrr} 5i & 2 & 3 & 50 & 75 \\ & & 10i & -50+15i & -75 \\ \hline & 2 & 3+10i & 15i & 0 \end{array}$$

$$\begin{array}{r|rrr} -5i & 2 & 3+10i & 15i \\ & & -10i & -15i \\ \hline & 2 & 3 & 0 \end{array}$$

The zero of $2x + 3$ is $x = -\frac{3}{2}$. The zeros of $f(x)$ are $x = -\frac{3}{2}$ and $x = \pm 5i$.

Alternate Solution

Since $x = \pm 5i$ are zeros of $f(x)$, $(x + 5i)(x - 5i) = x^2 + 25$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r} 2x + 3 \\ x^2 + 0x + 25 \overline{\smash{\big)}\, 2x^3 + 3x^2 + 50x + 75} \\ \underline{2x^3 + 0x^2 + 50x } \\ 3x^2 + 0x + 75 \\ \underline{3x^2 + 0x + 75} \\ 0 \end{array}$$

Thus, $f(x) = (x^2 + 25)(2x + 3)$ and the zeros of f are $x = \pm 5i$ and $x = -\frac{3}{2}$.

48. $f(x) = x^3 + x^2 + 9x + 9$

Since $3i$ is a zero, so is $-3i$.

$$\begin{array}{r|rrrr} 3i & 1 & 1 & 9 & 9 \\ & & 3i & -9+3i & -9 \\ \hline & 1 & 1+3i & 3i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3i & 1 & 1+3i & 3i \\ & & -3i & -3i \\ \hline & 1 & 1 & 0 \end{array}$$

The zero of $x + 1$ is $x = -1$. The zeros of f are $x = -1$ and $x = \pm 3i$.

49. $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

Since $2i$ is a zero, so is $-2i$.

$$\begin{array}{r|rrrrr} 2i & 2 & -1 & 7 & -4 & -4 \\ & & 4i & -8-2i & 4-2i & 4 \\ \hline & 2 & -1+4i & -1-2i & -2i & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2i & 2 & -1+4i & -1-2i & -2i \\ & & -4i & 2i & 2i \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

The zeros of $2x^2 - x - 1 = (2x + 1)(x - 1)$ are $x = -\frac{1}{2}$ and $x = 1$. The zeros of $f(x)$ are $x = \pm 2i$, $x = -\frac{1}{2}$, and $x = 1$.

Alternate Solution

Since $x = \pm 2i$ are zeros of $f(x)$, $(x + 2i)(x - 2i) = x^2 + 4$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r} 2x^2 - x - 1 \\ x^2 + 0x + 4 \overline{\smash{\big)}\, 2x^4 - x^3 + 7x^2 - 4x - 4} \\ \underline{2x^4 + 0x^3 + 8x^2 } \\ -x^3 - x^2 - 4x \\ \underline{-x^3 + 0x^2 - 4x } \\ -x^2 + 0x - 4 \\ \underline{-x^2 + 0x - 4} \\ 0 \end{array}$$

Thus, $f(x) = (x^2 + 4)(2x^2 - x - 1)$

$= (x + 2i)(x - 2i)(2x + 1)(x - 1)$

and the zeros of $f(x)$ are $x = \pm 2i$, $x = -\frac{1}{2}$, and $x = 1$.

50. $g(x) = x^3 - 7x^2 - x + 87$

Since $5 + 2i$ is a zero, so is $5 - 2i$.

$$\begin{array}{r|rrrr} 5+2i & 1 & -7 & -1 & 87 \\ & & 5+2i & -14+6i & -87 \\ \hline & 1 & -2+2i & -15+6i & 0 \end{array}$$

$$\begin{array}{r|rrr} 5-2i & 1 & -2+2i & -15+6i \\ & & 5-2i & 15-6i \\ \hline & 1 & 3 & 0 \end{array}$$

The zero of $x + 3$ is $x = -3$. The zeros of f are $x = -3, 5 \pm 2i$.

51. $g(x) = 4x^3 + 23x^2 + 34x - 10$

Since $-3 + i$ is a zero, so is $-3 - i$.

$$\begin{array}{r|rrrr} -3+i & 4 & 23 & 34 & -10 \\ & & -12+4i & -37-i & 10 \\ \hline & 4 & 11+4i & -3-i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3-i & 4 & 11+4i & -3-i \\ & & -12-4i & 3+i \\ \hline & 4 & -1 & 0 \end{array}$$

The zero of $4x - 1$ is $x = \frac{1}{4}$. The zeros of $g(x)$ are $x = -3 \pm i$ and $x = \frac{1}{4}$.

Alternate Solution

Since $-3 \pm i$ are zeros of $g(x)$,

$$\begin{aligned}[x - (-3 + i)][x - (-3 - i)] &= [(x + 3) - i][(x + 3) + i] \\ &= (x + 3)^2 - i^2 \\ &= x^2 + 6x + 10\end{aligned}$$

is a factor of $g(x)$. By long division we have:

$$\begin{array}{r} 4x - 1 \\ x^2 + 6x + 10\enclose{longdiv}{4x^3 + 23x^2 + 34x - 10} \\ \underline{4x^3 + 24x^2 + 40x} \\ -x^2 - 6x - 10 \\ \underline{-x^2 - 6x - 10} \\ 0 \end{array}$$

Thus, $g(x) = (x^2 + 6x + 10)(4x - 1)$ and the zeros of $g(x)$ are $x = -3 \pm i$ and $x = \frac{1}{4}$.

52. $h(x) = 3x^3 - 4x^2 + 8x + 8$

Since $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{array}{r|rrrr} 1-\sqrt{3}i & 3 & -4 & 8 & 8 \\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8 \\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i \\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i \\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x + 2$ is $x = -\frac{2}{3}$. The zeros of f are $x = -\frac{2}{3}, 1 \pm \sqrt{3}i$.

53. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$

Since $-3 + \sqrt{2}i$ is a zero, so is $-3 - \sqrt{2}i$, and

$$\begin{aligned}\left[x - \left(-3 + \sqrt{2}i\right)\right]\left[x - \left(-3 - \sqrt{2}i\right)\right] &= \left[(x + 3) - \sqrt{2}i\right]\left[(x + 3) + \sqrt{2}i\right] \\ &= (x + 3)^2 - \left(\sqrt{2}i\right)^2 \\ &= x^2 + 6x + 11\end{aligned}$$

is a factor of $f(x)$. By long division, we have:

$$\begin{array}{r} x^2 - 3x + 2 \\ x^2 + 6x + 11\enclose{longdiv}{x^4 + 3x^3 - 5x^2 - 21x + 22} \\ \underline{x^4 + 6x^3 + 11x^2} \\ -3x^3 - 16x^2 - 21x \\ \underline{-3x^3 - 18x^2 - 33x} \\ 2x^2 + 12x + 22 \\ \underline{2x^2 + 12x + 22} \\ 0 \end{array}$$

Thus,

$$\begin{aligned}f(x) &= (x^2 + 6x + 11)(x^2 - 3x + 2) \\ &= (x^2 + 6x + 11)(x - 1)(x - 2)\end{aligned}$$

and the zeros of f are $x = -3 \pm \sqrt{2}i$, $x = 1$, and $x = 2$.

54. $f(x) = x^3 + 4x^2 + 14x + 20$

Since $-1 - 3i$ is a zero, so is $-1 + 3i$.

$$\begin{array}{r|rrrr} -1-3i & 1 & 4 & 14 & 20 \\ & & -1-3i & -12-6i & -20 \\ \hline & 1 & 3-3i & 2-6i & 0 \end{array}$$

$$\begin{array}{r|rrr} -1+3i & 1 & 3-3i & 2-6i \\ & & -1+3i & -2+6i \\ \hline & 1 & 2 & 0 \end{array}$$

The zero of $x + 2$ is $x = -2$.

The zeros of f are $x = -2, -1 \pm 3i$.

55. $f(x) = x^2 + 25$

$= (x + 5i)(x - 5i)$

The zeros of $f(x)$ are $x = \pm 5i$.

56. $f(x) = x^2 - x + 56$

By the Quadratic Formula, the zeros of $f(x)$ are

$$x = \frac{1 \pm \sqrt{1 - 224}}{2} = \frac{1 \pm \sqrt{223}i}{2}.$$

$$f(x) = \left(x - \frac{1 - \sqrt{223}i}{2}\right)\left(x - \frac{1 + \sqrt{223}i}{2}\right)$$

57. $h(x) = x^2 - 4x + 1$

By the Quadratic Formula, the zeros of $h(x)$ are

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}.$$

$$h(x) = \left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right]$$
$$= \left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right)$$

58. $g(x) = x^2 + 10x + 23$

By the Quadratic Formula, the zeros of $f(x)$ are

$$x = \frac{-10 \pm \sqrt{100 - 92}}{2} = \frac{-10 \pm \sqrt{8}}{2} = -5 \pm \sqrt{2}.$$

$$g(x) = \left(x + 5 + \sqrt{2}\right)\left(x + 5 - \sqrt{2}\right)$$

59. $f(x) = x^4 - 81$

$= (x^2 - 9)(x^2 + 9)$

$= (x + 3)(x - 3)(x + 3i)(x - 3i)$

The zeros of $f(x)$ are $x = \pm 3$ and $x = \pm 3i$.

60. $f(y) = y^4 - 625$

$= (y^2 + 25)(y^2 - 25)$

Zeros: $y = \pm 5, \pm 5i$

$f(y) = (y + 5)(y - 5)(y + 5i)(y - 5i)$

61. $f(z) = z^2 - 2z + 2$

By the Quadratic Formula, the zeros of $f(z)$ are

$$z = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i.$$

$$f(z) = [z - (1 + i)][z - (1 - i)]$$
$$= (z - 1 - i)(z - 1 + i)$$

62. $h(x) = x^3 - 3x^2 + 4x - 2$

Possible rational zeros: $\pm 1, \pm 2$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 4 & -2 \\ & & 1 & -2 & 2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 2$

are $x = \dfrac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i.$

Zeros: $x = 1, 1 \pm i$

$h(x) = (x - 1)(x - 1 - i)(x - 1 + i)$

63. $g(x) = x^3 - 6x^2 + 13x - 10$

Possible rational zeros: $\pm 1, \pm 2, \pm 5, \pm 10$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 13 & -10 \\ & & 2 & -8 & 10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 4x + 5$ are

$$x = \frac{4 \pm \sqrt{16 - 20}}{2} = 2 \pm i.$$

The zeros of $g(x)$ are $x = 2$ and $x = 2 \pm i$.

$$g(x) = (x - 2)[x - (2 + i)][x - (2 - i)]$$
$$= (x - 2)(x - 2 - i)(x - 2 + i)$$

64. $f(x) = x^3 - 2x^2 - 11x + 52$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 13, \pm 26$

$$\begin{array}{r|rrrr} -4 & 1 & -2 & -11 & 52 \\ & & -4 & 24 & -52 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 6x + 13$ are $x = \dfrac{6 \pm \sqrt{36 - 52}}{2} = 3 \pm 2i$.

Zeros: $x = -4, 3 \pm 2i$

$f(x) = (x + 4)(x - 3 - 2i)(x - 3 + 2i)$

65. $h(x) = x^3 - x + 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -1 & 6 \\ & & -2 & 4 & -6 \\ \hline & 1 & -2 & 3 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 3$ are

$x = \dfrac{2 \pm \sqrt{4 - 12}}{2} = 1 \pm \sqrt{2}\,i.$

The zeros of $h(x)$ are $x = -2$ and $x = 1 \pm \sqrt{2}\,i$.

$$\begin{aligned} h(x) &= [x - (-2)]\left[x - \left(1 + \sqrt{2}\,i\right)\right]\left[x - \left(1 - \sqrt{2}\,i\right)\right] \\ &= (x + 2)\left(x - 1 - \sqrt{2}\,i\right)\left(x - 1 + \sqrt{2}\,i\right) \end{aligned}$$

66. $h(x) = x^3 + 9x^2 + 27x + 35$

Possible rational zeros: $\pm 1, \pm 5, \pm 7, \pm 35$

$$\begin{array}{r|rrrr} -5 & 1 & 9 & 27 & 35 \\ & & -5 & -20 & -35 \\ \hline & 1 & 4 & 7 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 + 4x + 7$

are $x = \dfrac{-4 \pm \sqrt{16 - 28}}{2} = -2 \pm \sqrt{3}i.$

Zeros: $-5, -2 \pm \sqrt{3}i$

$h(x) = (x + 5)\left(x + 2 + \sqrt{3}i\right)\left(x + 2 - \sqrt{3}i\right)$

67. $f(x) = 5x^3 - 9x^2 + 28x + 6$

Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{6}{5}$

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

By the Quadratic Formula, the zeros of
$5x^2 - 10x + 30 = 5(x^2 - 2x + 6)$ are

$x = \dfrac{2 \pm \sqrt{4 - 24}}{2} = 1 \pm \sqrt{5}\,i.$

The zeros of $f(x)$ are $x = -\frac{1}{5}$ and $x = 1 \pm \sqrt{5}\,i$.

$$\begin{aligned} f(x) &= \left[x - \left(-\tfrac{1}{5}\right)\right](5)\left[x - \left(1 + \sqrt{5}\,i\right)\right]\left[x - \left(1 - \sqrt{5}\,i\right)\right] \\ &= (5x + 1)\left(x - 1 - \sqrt{5}\,i\right)\left(x - 1 + \sqrt{5}\,i\right) \end{aligned}$$

68. $g(x) = 3x^3 - 4x^2 + 8x + 8$

Possible rational zeros:
$\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

$$\begin{array}{r|rrrr} -\frac{2}{3} & 3 & -4 & 8 & 8 \\ & & -2 & 4 & -8 \\ \hline & 3 & -6 & 12 & 0 \end{array}$$

By the Quadratic Formula, the zeros of
$3x^2 - 6x + 12 = 3(x^2 - 2x + 4)$ are

$x = \dfrac{2 \pm \sqrt{4 - 16}}{2} = 1 \pm \sqrt{3}i.$

Zeros: $x = -\frac{2}{3}, 1 \pm \sqrt{3}i$

$g(x) = (3x + 2)\left(x - 1 + \sqrt{3}i\right)\left(x - 1 - \sqrt{3}i\right)$

69. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 4 & -8 \\ & & 2 & 0 & 8 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$g(x) = (x - 2)(x - 2)(x^2 + 4) = (x - 2)^2(x + 2i)(x - 2i)$

The zeros of $g(x)$ are 2 and $\pm 2i$.

70. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

Possible rational zeros: $\pm 1, \pm 3, \pm 9$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline & 1 & 3 & 1 & 3 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

The zeros of $x^2 + 1$ are $x = \pm i$.

Zeros: $x = -3, \pm i$

$h(x) = (x + 3)^2(x + i)(x - i)$

71. $f(x) = x^4 + 10x^2 + 9$

$= (x^2 + 1)(x^2 + 9)$

$= (x + i)(x - i)(x + 3i)(x - 3i)$

The zeros of $f(x)$ are $x = \pm i$ and $x = \pm 3i$.

72. $f(x) = x^4 + 29x^2 + 100$

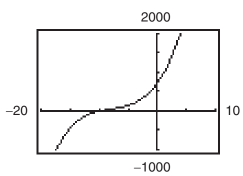
$= (x^2 + 25)(x^2 + 4)$

Zeros: $x = \pm 2i, \pm 5i$

$f(x) = (x + 2i)(x - 2i)(x + 5i)(x - 5i)$

73. $f(x) = x^3 + 24x^2 + 214x + 740$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 37,$
$\pm 74, \pm 148, \pm 185, \pm 370, \pm 740$



Based on the graph, try $x = -10$.

$$\begin{array}{r|rrrr} -10 & 1 & 24 & 214 & 740 \\ & & -10 & -140 & -740 \\ \hline & 1 & 14 & 74 & 0 \end{array}$$

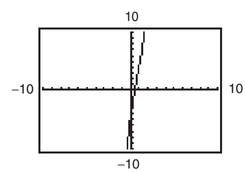
By the Quadratic Formula, the zeros of $x^2 + 14x + 74$ are

$$x = \frac{-14 \pm \sqrt{196 - 296}}{2} = -7 \pm 5i.$$

The zeros of $f(x)$ are $x = -10$ and $x = -7 \pm 5i$.

74. $f(s) = 2s^3 - 5s^2 + 12s - 5$

Possible rational zeros: $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$



Based on the graph, try $s = \frac{1}{2}$.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -5 & 12 & -5 \\ & & 1 & -2 & 5 \\ \hline & 2 & -4 & 10 & 0 \end{array}$$

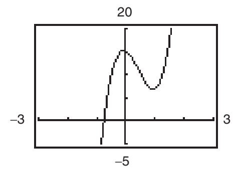
By the Quadratic Formula, the zeros of $2(s^2 - 2s + 5)$ are

$$s = \frac{2 \pm \sqrt{4 - 20}}{2} = 1 \pm 2i.$$

The zeros of $f(s)$ are $s = \frac{1}{2}$ and $s = 1 \pm 2i$.

75. $f(x) = 16x^3 - 20x^2 - 4x + 15$

Possible rational zeros:
$\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}, \pm\frac{15}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{5}{16}, \pm\frac{15}{16}$



Based on the graph, try $x = -\frac{3}{4}$.

$$\begin{array}{r|rrrr} -\frac{3}{4} & 16 & -20 & -4 & 15 \\ & & -12 & 24 & -15 \\ \hline & 16 & -32 & 20 & 0 \end{array}$$

By the Quadratic Formula, the zeros of

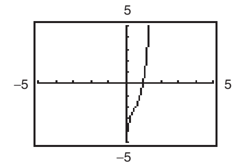
$16x^2 - 32x + 20 = 4(4x^2 - 8x + 5)$ are

$$x = \frac{8 \pm \sqrt{64 - 80}}{8} = 1 \pm \frac{1}{2}i.$$

The zeros of $f(x)$ are $x = -\frac{3}{4}$ and $x = 1 \pm \frac{1}{2}i$.

76. $f(x) = 9x^3 - 15x^2 + 11x - 5$

Possible rational zeros: $\pm1, \pm5, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{9}, \pm\frac{5}{9}$



Based on the graph, try $x = 1$.

$$\begin{array}{r|rrrr} 1 & 9 & -15 & 11 & -5 \\ & & 9 & -6 & 5 \\ \hline & 9 & -6 & 5 & 0 \end{array}$$

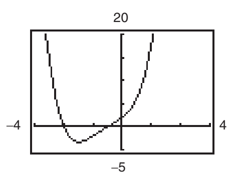
By the Quadratic Formula, the zeros of $9x^2 - 6x + 5$ are

$$x = \frac{6 \pm \sqrt{36 - 180}}{18} = \frac{1}{3} \pm \frac{2}{3}i.$$

The zeros of $f(x)$ are $x = 1$ and $x = \frac{1}{3} \pm \frac{2}{3}i$.

77. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

Possible rational zeros: $\pm1, \pm2, \pm\frac{1}{2}$



Based on the graph, try $x = -2$ and $x = -\frac{1}{2}$.

$$\begin{array}{r|rrrrr} -2 & 2 & 5 & 4 & 5 & 2 \\ & & -4 & -2 & -4 & -2 \\ \hline & 2 & 1 & 2 & 1 & 0 \end{array}$$

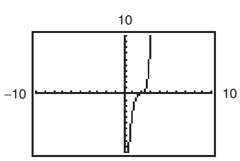
$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 1 & 2 & 1 \\ & & -1 & 0 & -1 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$

The zeros of $2x^2 + 2 = 2(x^2 + 1)$ are $x = \pm i$.

The zeros of $f(x)$ are $x = -2$, $x = -\frac{1}{2}$, and $x = \pm i$.

78. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16, \pm32$



Based on the graph, try $x = 2$.

$$\begin{array}{r|rrrrrr} 2 & 1 & -8 & 28 & -56 & 64 & -32 \\ & & 2 & -12 & 32 & -48 & 32 \\ \hline & 1 & -6 & 16 & -24 & 16 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 16 & -24 & 16 \\ & & 2 & -8 & 16 & -16 \\ \hline & 1 & -4 & 8 & -8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 8 & -8 \\ & & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 4$ are

$$x = \frac{2 \pm \sqrt{4 - 16}}{2} = 1 \pm \sqrt{3}i.$$

The zeros of $g(x)$ are $x = 2$ and $x = 1 \pm \sqrt{3}i$.

79. $g(x) = 5x^5 + 10x = 5x(x^4 + 2)$

Let $f(x) = x^4 + 2$.

Sign variations: 0, positive zeros: 0

$f(-x) = x^4 + 2$

Sign variations: 0, negative zeros: 0

80. $h(x) = 4x^2 - 8x + 3$

Sign variations: 2, positive zeros: 2 or 0

$h(-x) = 4x^2 + 8x + 3$

Sign variations: 0, negative zeros: 0

81. $h(x) = 3x^4 + 2x^2 + 1$

Sign variations: 0, positive zeros: 0

$h(-x) = 3x^4 + 2x^2 + 1$

Sign variations: 0, negative zeros: 0

82. $h(x) = 2x^4 - 3x + 2$

Sign variations: 2, positive zeros: 2 or 0

$h(-x) = 2x^4 + 3x + 2$

Sign variations: 0, negative zeros: 0

83. $g(x) = 2x^3 - 3x^2 - 3$

Sign variations: 1, positive zeros: 1

$g(-x) = -2x^3 - 3x^2 - 3$

Sign variations: 0, negative zeros: 0

84. $f(x) = 4x^3 - 3x^2 + 2x - 1$

Sign variations: 3, positive zeros: 3 or 1

$f(-x) = -4x^3 - 3x^2 - 2x - 1$

Sign variations: 0, negative zeros: 0

85. $f(x) = -5x^3 + x^2 - x + 5$

Sign variations: 3, positive zeros: 3 or 1

$f(-x) = 5x^3 + x^2 + x + 5$

Sign variations: 0, negative zeros: 0

86. $f(x) = 3x^3 + 2x^2 + x + 3$

Sign variations: 0, positive zeros: 0

$f(-x) = -3x^3 + 2x^2 - x + 3$

Sign variations: 3, negative zeros: 3 or 1

87. $f(x) = x^4 - 4x^3 + 15$

(a) $\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 0 & 15 \\ & & 4 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 15 \end{array}$

4 is an upper bound.

(b) $\begin{array}{r|rrrrr} -1 & 1 & -4 & 0 & 0 & 15 \\ & & -1 & 5 & -5 & 5 \\ \hline & 1 & -5 & 5 & -5 & 20 \end{array}$

-1 is a lower bound.

88. $f(x) = 2x^3 - 3x^2 - 12x + 8$

(a) $\begin{array}{r|rrrr} 4 & 2 & -3 & -12 & 8 \\ & & 8 & 20 & 32 \\ \hline & 2 & 5 & 8 & 40 \end{array}$

4 is an upper bound.

(b) $\begin{array}{r|rrrr} -3 & 2 & -3 & -12 & 8 \\ & & -6 & 27 & -45 \\ \hline & 2 & -9 & 15 & -37 \end{array}$

-3 is a lower bound.

89. $f(x) = x^4 - 4x^3 + 16x - 16$

(a) $\begin{array}{r|rrrrr} 5 & 1 & -4 & 0 & 16 & -16 \\ & & 5 & 5 & 25 & 205 \\ \hline & 1 & 1 & 5 & 41 & 189 \end{array}$

5 is an upper bound.

(b) $\begin{array}{r|rrrrr} -3 & 1 & -4 & 0 & 16 & -16 \\ & & -3 & 21 & -63 & 141 \\ \hline & 1 & -7 & 21 & -47 & 125 \end{array}$

-3 is a lower bound.

90. $f(x) = 2x^4 - 8x + 3$

(a) $\begin{array}{r|rrrrr} 3 & 2 & 0 & 0 & -8 & 3 \\ & & 6 & 18 & 54 & 138 \\ \hline & 2 & 6 & 18 & 46 & 141 \end{array}$

3 is an upper bound.

(b) $\begin{array}{r|rrrrr} -4 & 2 & 0 & 0 & -8 & 3 \\ & & -8 & 32 & -128 & 544 \\ \hline & 2 & -8 & 32 & -136 & 547 \end{array}$

-3 is a lower bound.

91. $f(x) = 4x^3 - 3x - 1$

Possible rational zeros: $\pm 1, \ \pm\frac{1}{2}, \ \pm\frac{1}{4}$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & -1 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

$$4x^3 - 3x - 1 = (x - 1)(4x^2 + 4x + 1)$$
$$= (x - 1)(2x + 1)^2$$

Thus, the zeros are 1 and $-\frac{1}{2}$.

93. $f(y) = 4y^3 + 3y^2 + 8y + 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 4 & 3 & 8 & 6 \\ & & -3 & 0 & -6 \\ \hline & 4 & 0 & 8 & 0 \end{array}$$

$$4y^3 + 3y^2 + 8y + 6 = \left(y + \tfrac{3}{4}\right)(4y^2 + 8)$$
$$= \left(y + \tfrac{3}{4}\right)4(y^2 + 2)$$
$$= (4y + 3)(y^2 + 2)$$

Thus, the only real zero is $-\frac{3}{4}$.

95. $P(x) = x^4 - \frac{25}{4}x^2 + 9$

$$= \tfrac{1}{4}(4x^4 - 25x^2 + 36)$$
$$= \tfrac{1}{4}(4x^2 - 9)(x^2 - 4)$$
$$= \tfrac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2)$$

The rational zeros are $\pm\frac{3}{2}$ and ± 2.

97. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

$$= \tfrac{1}{4}(4x^3 - x^2 - 4x + 1)$$
$$= \tfrac{1}{4}[x^2(4x - 1) - 1(4x - 1)]$$
$$= \tfrac{1}{4}(4x - 1)(x^2 - 1)$$
$$= \tfrac{1}{4}(4x - 1)(x + 1)(x - 1)$$

The rational zeros are $\frac{1}{4}$ and ± 1.

92. $f(z) = 12z^3 - 4z^2 - 27z + 9$

Possible rational zeros: $\pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$

$$\begin{array}{r|rrrr} \frac{3}{2} & 12 & -4 & -27 & 9 \\ & & 18 & 21 & -9 \\ \hline & 12 & 14 & -6 & 0 \end{array}$$

$$f(z) = 2\left(z - \tfrac{3}{2}\right)(6z^2 + 7z - 3)$$
$$= (2z - 3)(3z - 1)(2z + 3)$$

Real zeros: $-\frac{3}{2}, \frac{1}{3}, \frac{3}{2}$

94. $g(x) = 3x^3 - 2x^2 + 15x - 10$

Possible rational zeros: $\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & 15 & -10 \\ & & 2 & 0 & 10 \\ \hline & 3 & 0 & 15 & 0 \end{array}$$

$$g(x) = \left(x - \tfrac{2}{3}\right)(3x^2 + 15) = (3x - 2)(x^2 + 5)$$

Thus, the only real zero is $\frac{2}{3}$.

96. $f(x) = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$f(x) = \tfrac{1}{2}(x - 4)(2x^2 + 5x - 3) = \tfrac{1}{2}(x - 4)(2x - 1)(x + 3)$$

The rational zeros are $-3, \frac{1}{2}$, and 4.

98. $f(z) = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

$$\begin{array}{r|rrrr} -2 & 6 & 11 & -3 & -2 \\ & & -12 & 2 & 2 \\ \hline & 6 & -1 & -1 & 0 \end{array}$$

$$f(x) = \tfrac{1}{6}(x + 2)(6x^2 - x - 1)$$
$$= \tfrac{1}{6}(x + 2)(3x + 1)(2x - 1)$$

Rational zeros: $-2, -\frac{1}{3}, \frac{1}{2}$

99. $f(x) = x^3 - 1 = (x - 1)(x^2 + x + 1)$

Rational zeros: 1 $(x = 1)$

Irrational zeros: 0

Matches (d).

100. $f(x) = x^3 - 2$

$= \left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + \sqrt[3]{4}\right)$

Rational zeros: 0

Irrational zeros: 1 $\left(x = \sqrt[3]{2}\right)$

Matches (a).

101. $f(x) = x^3 - x = x(x + 1)(x - 1)$

Rational zeros: 3 $(x = 0, \pm 1)$

Irrational zeros: 0

Matches (b).

102. $f(x) = x^3 - 2x$

$= x(x^2 - 2)$

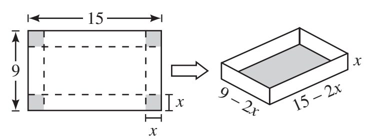
$= x\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

Rational zeros: 1 $(x = 0)$

Irrational zeros: 2 $\left(x = \pm\sqrt{2}\right)$

Matches (c).

103. (a)

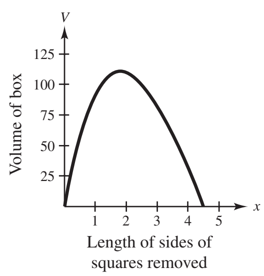


(b) $V = l \cdot w \cdot h = (15 - 2x)(9 - 2x)x$

$= x(9 - 2x)(15 - 2x)$

Since length, width, and height must be positive, we have $0 < x < \frac{9}{2}$ for the domain.

(c)



The volume is maximum when $x \approx 1.82$.

The dimensions are: length $\approx 15 - 2(1.82) = 11.36$

width $\approx 9 - 2(1.82) = 5.36$

height $= x \approx 1.82$

1.82 cm $\times$ 5.36 cm $\times$ 11.36 cm

(d) $56 = x(9 - 2x)(15 - 2x)$

$56 = 135x - 48x^2 + 4x^3$

$0 = 4x^3 - 48x^2 + 135x - 56$

The zeros of this polynomial are $\frac{1}{2}$, $\frac{7}{2}$, and 8.
x cannot equal 8 since it is not in the domain of V.
[The length cannot equal -1 and the width cannot equal -7. The product of $(8)(-1)(-7) = 56$ so it showed up as an extraneous solution.]

Thus, the volume is 56 cubic centimeters when $x = \frac{1}{2}$ centimeter or $x = \frac{7}{2}$ centimeters.

104. (a) Combined length and width:

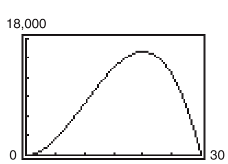
$4x + y = 120 \implies y = 120 - 4x$

$\text{Volume} = l \cdot w \cdot h = x^2y$

$= x^2(120 - 4x)$

$= 4x^2(30 - x)$

(b)



Dimensions with maximum volume:

20 in. $\times$ 20 in. $\times$ 40 in.

(c) $13{,}500 = 4x^2(30 - x)$

$4x^3 - 120x^2 + 13{,}500 = 0$

$x^3 - 30x^2 + 3375 = 0$

$$\begin{array}{r|rrrr} 15 & 1 & -30 & 0 & 3375 \\ & & 15 & -225 & -3375 \\ \hline & 1 & -15 & -225 & 0 \end{array}$$

$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic Formula, $x = 15, \dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

105. $$P = -76x^3 + 4830x^2 - 320{,}000,\ 0 \le x \le 60$$
$$2{,}500{,}000 = -76x^3 + 4830x^2 - 320{,}000$$
$$76x^3 - 4830x^2 + 2{,}820{,}000 = 0$$

The zeros of this equation are $x \approx 46.1$, $x \approx 38.4$, and $x \approx -21.0$. Since $0 \le x \le 60$, we disregard $x \approx -21.0$. The smaller remaining solution is $x \approx 38.4$. The advertising expense is \$384,000.

106. $$P = -45x^3 + 2500x^2 - 275{,}000$$
$$800{,}000 = -45x^3 + 2500x^2 - 275{,}000$$
$$0 = 45x^3 - 2500x^2 + 1{,}075{,}000$$
$$0 = 9x^3 - 500x^2 + 215{,}000$$

The zeros of this equation are $x \approx -18.0$, $x \approx 31.5$, and $x \approx 42.0$. Because $0 \le x \le 50$, disregard $x \approx -18.02$. The smaller remaining solution is $x \approx 31.5$, or an advertising expense of \$315,000.

107. (a) Current bin: $V = 2 \times 3 \times 4 = 24$ cubic feet

New bin: $V = 5(24) = 120$ cubic feet

$$V = (2 + x)(3 + x)(4 + x) = 120$$

(b) $x^3 + 9x^2 + 26x + 24 = 120$

$x^3 + 9x^2 + 26x - 96 = 0$

The only real zero of this polynomial is $x = 2$. All the dimensions should be increased by 2 feet, so the new bin will have dimensions of 4 feet by 5 feet by 6 feet.

108. (a) $A = (250 + x)(160 + x) = (1.5)(160)(250)$

$= 60{,}000$

(b) $60{,}000 = x^2 + 410x + 40{,}000$

$0 = x^2 + 410x - 20{,}000$

$$x = \frac{-410 \pm \sqrt{410^2 - (4)(1)(-20{,}000)}}{2(1)}$$
$$= \frac{-410 \pm \sqrt{248{,}100}}{2}$$

x must be positive, so

$$x = \frac{-410 + \sqrt{248{,}100}}{2}$$
$$\approx 44.05.$$

The new length is $250 + 44.05 = 294.05$ ft and the new width is $160 + 44.05 = 204.05$ ft, so the new dimensions are $204.05 \text{ ft} \times 294.05$ ft.

(c) $A = (250 + 2x)(160 + x) = 60{,}000$

$2x^2 + 570x - 20{,}000 = 0$

$$x = \frac{-570 \pm \sqrt{570^2 - (4)(2)(-20{,}000)}}{2(2)}$$

x must be positive, so

$$x = \frac{-570 + \sqrt{484{,}900}}{4} \approx 31.6.$$

The new length is $250 + 2(31.6) = 313.2$ ft and the new width is $160 + (31.6) = 191.6$ ft, so the new dimensions are $191.6 \text{ ft} \times 313.2$ ft.

109. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x + 30}\right),\ x \ge 1$

C is minimum when $3x^3 - 40x^2 - 2400x - 36000 = 0$.

The only real zero is $x \approx 40$ or 4000 units.

110. $h(t) = -16t^2 + 48t + 6$

Let $h = 64$ and solve for t.

$64 = -16t^2 + 48t + 6$

$16t^2 - 48t + 58 = 0$

By the Quadratic Formula we have $t = \dfrac{48 \pm i\sqrt{1408}}{32}$.

Since the equation yields only imaginary zeros, it is *not* possible for the ball to have reached a height of 64 feet.

111. $P = R - C = xp - C$

$= x(140 - 0.0001x) - (80x + 150{,}000)$

$= -0.0001x^2 + 60x - 150{,}000$

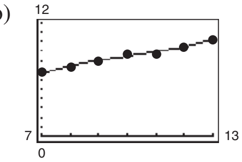
$9{,}000{,}000 = -0.0001x^2 + 60x - 150{,}000$

Thus, $0 = 0.0001x^2 - 60x + 9{,}150{,}000.$

$$x = \frac{60 \pm \sqrt{-60}}{0.0002} = 300{,}000 \pm 10{,}000\sqrt{15}\,i$$

Since the solutions are both complex, it is not possible to determine a price p that would yield a profit of 9 million dollars.

112. (a) $A \approx 0.0167t^3 - 0.508t^2 + 5.60t - 13.4$

(b) 

The model is a good fit to the actual data.

(c) $A = 8.5$ when $t \approx 10$ which corresponds to the year 2000.

(d) $A = 9$ when $t \approx 11$ which corresponds to the year 2001.

(e) Yes. The degree of A is odd and the leading coefficient is positive, so as x increases, A will increase. This implies that attendance will continue to grow.

113. False. The most nonreal complex zeros it can have is two and the Linear Factorization Theorem guarantees that there are 3 linear factors, so one zero must be real.

114. False. f does not have real coefficients.

115. $g(x) = -f(x)$. This function would have the same zeros as $f(x)$ so r_1, r_2, and r_3 are also zeros of $g(x)$.

116. $g(x) = 3f(x)$. This function has the same zeros as f because it is a vertical stretch of f. The zeros of g are r_1, r_2, and r_3.

117. $g(x) = f(x - 5)$. The graph of $g(x)$ is a horizontal shift of the graph of $f(x)$ five units to the right so the zeros of $g(x)$ are $5 + r_1$, $5 + r_2$, and $5 + r_3$.

118. $g(x) = f(2x)$. Note that x is a zero of g if and only if $2x$ is a zero of f. The zeros of g are $\frac{r_1}{2}, \frac{r_2}{2}$, and $\frac{r_3}{2}$.

119. $g(x) = 3 + f(x)$. Since $g(x)$ is a vertical shift of the graph of $f(x)$, the zeros of $g(x)$ cannot be determined.

120. $g(x) = f(-x)$. Note that x is a zero of g if and only if $-x$ is a zero of f. The zeros of g are $-r_1$, $-r_2$, and $-r_3$.

121. $f(x) = x^4 - 4x^2 + k$

$$x^2 = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(k)}}{2(1)} = \frac{4 \pm 2\sqrt{4 - k}}{2} = 2 \pm \sqrt{4 - k}$$

$$x = \pm\sqrt{2 \pm \sqrt{4 - k}}$$

(a) For there to be four distinct real roots, both $4 - k$ and $2 \pm \sqrt{4 - k}$ must be positive. This occurs when $0 < k < 4$. Thus, some possible k-values are $k = 1$, $k = 2$, $k = 3$, $k = \frac{1}{2}$, $k = \sqrt{2}$, etc.

(b) For there to be two real roots, each of multiplicity 2, $4 - k$ must equal zero. Thus, $k = 4$.

(c) For there to be two real zeros and two complex zeros, $2 + \sqrt{4 - k}$ must be positive and $2 - \sqrt{4 - k}$ must be negative. This occurs when $k < 0$. Thus, some possible k-values are $k = -1$, $k = -2$, $k = -\frac{1}{2}$, etc.

(d) For there to be four complex zeros, $2 \pm \sqrt{4 - k}$ must be nonreal. This occurs when $k > 4$. Some possible k-values are $k = 5$, $k = 6$, $k = 7.4$, etc.

122. (a) $g(x) = f(x - 2)$

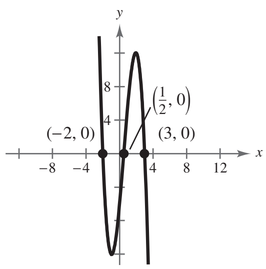
No. This function is a horizontal shift of $f(x)$. Note that x is a zero of g if and only if $x - 2$ is a zero of f; the number of real and complex zeros is not affected by a horizontal shift.

(b) $g(x) = f(2x)$

No. Since x is a zero of g if and only if $2x$ is a zero of f, the number of real and complex zeros of g is the same as the number of real and complex zeros of f.

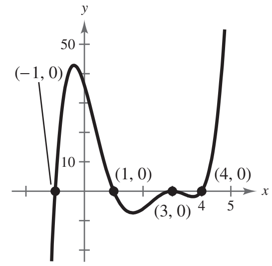
123. Zeros: $-2, \frac{1}{2}, 3$

$$f(x) = -(x+2)(2x-1)(x-3)$$
$$= -2x^3 + 3x^2 + 11x - 6$$



Any nonzero scalar multiple of f would have the same three zeros. Let $g(x) = af(x)$, $a > 0$. There are infinitely many possible functions for f.

124.



125. Answers will vary. Some of the factoring techniques are:

1. Factor out the greatest common factor.
2. Use special product formulas.

   $a^2 - b^2 = (a+b)(a-b)$

   $a^2 + 2ab + b^2 = (a+b)^2$

   $a^2 - 2ab + b^2 = (a-b)^2$

   $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

   $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
3. Factor by grouping, if possible.
4. Factor general trinomials with binomial factors by "guess-and-test" or by the grouping method.
5. Use the Rational Zero Test together with synthetic division to factor a polynomial.
6. Use Descartes's Rule of Signs to determine the number of real zeros. Then find any zeros and use them to factor the polynomial.
7. Find any upper and lower bounds for the real zeros to eliminate some of the possible rational zeros. Then test the remaining candidates by synthetic division and use any zeros to factor the polynomial.

126. (a) Zeros of $f(x)$: $-2, 1, 4$

(b) The graph touches the x-axis at $x = 1$

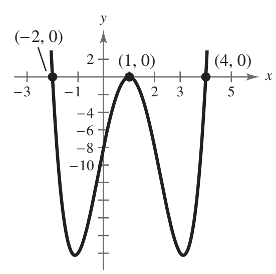
(c) The least possible degree of the function is 4 because there are at least 4 real zeros (1 is repeated) and a function can have at most the number of real zeros equal to the degree of the function. The degree cannot be odd by the definition of multiplicity.

(d) The leading coefficient of f is positive. From the information in the table, you can conclude that the graph will eventually rise to the left and to the right.

(e) Answers may vary. One possibility is:

$$f(x) = (x-1)^2(x-(-2))(x-4)$$
$$= (x-1)^2(x+2)(x-4)$$
$$= (x^2 - 2x + 1)(x^2 - 2x - 8)$$
$$= x^4 - 4x^3 - 3x^2 + 14x - 8$$

(f)



127. (a) $f(x) = \left(x - \sqrt{b}\,i\right)\left(x + \sqrt{b}\,i\right) = x^2 + b$

(b) $f(x) = [x - (a+bi)][x - (a-bi)]$

$$= [(x-a) - bi][(x-a) + bi]$$
$$= (x-a)^2 - (bi)^2$$
$$= x^2 - 2ax + a^2 + b^2$$

128. (a) $f(x)$ cannot have this graph since it also has a zero at $x = 0$.

(b) $g(x)$ cannot have this graph since it is a quadratic function. Its graph is a parabola.

(c) $h(x)$ is the correct function. It has two real zeros, $x = 2$ and $x = 3.5$, and it has a degree of four, needed to yield three turning points.

(d) $k(x)$ cannot have this graph since it also has a zero at $x = -1$. In addition, since it is only of degree three, it would have at most two turning points.

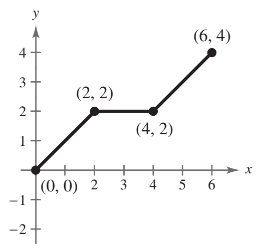
129. $(-3 + 6i) - (8 - 3i) = -3 + 6i - 8 + 3i = -11 + 9i$

130. $(12 - 5i) + 16i = 12 + 11i$

131. $(6 - 2i)(1 + 7i) = 6 + 42i - 2i - 14i^2 = 20 + 40i$

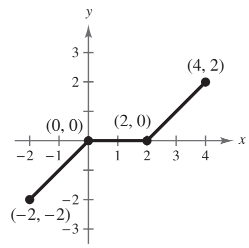
132. $(9 - 5i)(9 + 5i) = 81 - 25i^2 = 81 + 25 = 106$

133. $g(x) = f(x - 2)$



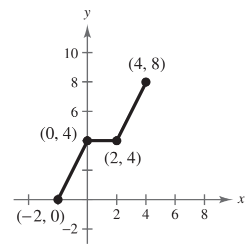
Horizontal shift two units to the right

134. $g(x) = f(x) - 2$



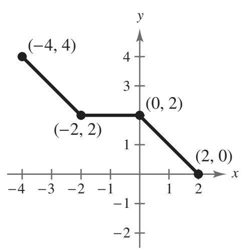
Vertical shift two units downward

135. $g(x) = 2f(x)$



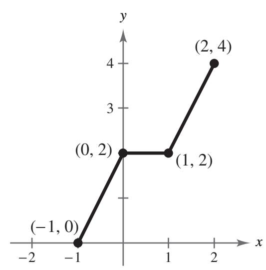
Vertical stretch (each y-value is multiplied by 2)

136. $g(x) = f(-x)$



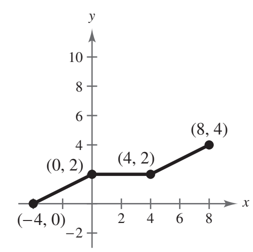
Reflection in the y-axis

137. $g(x) = f(2x)$



Horizontal shrink (each x-value is multiplied by $\frac{1}{2}$)

138. $g(x) = f\left(\frac{1}{2}x\right)$



Horizontal stretch (each x-value is multiplied by 2)

Section 2.6 Rational Functions

- You should know the following basic facts about rational functions.
  (a) A function of the form $f(x) = N(x)/D(x)$, $D(x) \neq 0$, where $N(x)$ and $D(x)$ are polynomials, is called a rational function.
  (b) The domain of a rational function is the set of all real numbers except those which make the denominator zero.
  (c) If $f(x) = N(x)/D(x)$ is in reduced form, and a is a value such that $D(a) = 0$, then the line $x = a$ is a vertical asymptote of the graph of f. $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$.
  (d) The line $y = b$ is a horizontal asymptote of the graph of f if $f(x) \longrightarrow b$ as $x \longrightarrow \infty$ or $x \longrightarrow -\infty$.
  (e) Let $f(x) = \dfrac{N(x)}{D(x)} = \dfrac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$ where $N(x)$ and $D(x)$ have no common factors.
    1. If $n < m$, then the x-axis $(y = 0)$ is a horizontal asymptote.
    2. If $n = m$, then $y = \dfrac{a_n}{b_m}$ is a horizontal asymptote.
    3. If $n > m$, then there are no horizontal asymptotes.

Vocabulary Check

1. rational functions **2.** vertical asymptote **3.** horizontal asymptote **4.** slant asymptote

1. $f(x) = \dfrac{1}{x-1}$

(a)

x	$f(x)$
0.5	-2
0.9	-10
0.99	-100
0.999	-1000

x	$f(x)$
1.5	2
1.1	10
1.01	100
1.001	1000

x	$f(x)$
5	0.25
10	$0.\overline{1}$
100	$0.\overline{01}$
1000	$0.\overline{001}$

(b) The zero of the denominator is $x = 1$, so $x = 1$ is a vertical asymptote. The degree of the numerator is less than the degree of the denominator so the x-axis, or $y = 0$, is a horizontal asymptote.

(c) The domain is all real numbers except $x = 1$.

2. $f(x) = \dfrac{5x}{x-1}$

(a)

x	$f(x)$
0.5	-5
0.9	-45
0.99	-495
0.999	-4995

x	$f(x)$
1.5	15
1.1	55
1.01	505
1.001	5005

x	$f(x)$
5	6.25
10	$5.\overline{55}$
100	$5.\overline{05}$
1000	$5.\overline{005}$

(b) The zero of the denominator is $x = 1$, so $x = 1$ is a vertical asymptote. The degree of the numerator is equal to the degree of the denominator, so the line $y = \frac{5}{1} = 5$ is a horizontal asymptote.

(c) The domain is all real numbers except $x = 1$.

3. $f(x) = \dfrac{3x^2}{x^2-1}$

(a)

x	$f(x)$
0.5	-1
0.9	-12.79
0.99	-147.8
0.999	-1498

x	$f(x)$
1.5	5.4
1.1	17.29
1.01	152.3
1.001	1502

x	$f(x)$
5	3.125
10	$3.\overline{03}$
100	$3.\overline{0003}$
1000	3

(b) The zeros of the denominator are $x = \pm 1$ so both $x = 1$ and $x = -1$ are vertical asymptotes. Since the degree of the numerator equals the degree of the denominator, $y = \frac{3}{1} = 3$ is a horizontal asymptote.

(c) The domain is all real numbers except $x = \pm 1$.

4. $f(x) = \dfrac{4x}{x^2-1}$

(a)

x	$f(x)$
0.5	$-2.\overline{66}$
0.9	-18.95
0.99	-199
0.999	-1999

x	$f(x)$
1.5	4.8
1.1	20.95
1.01	201
1.001	2001

x	$f(x)$
5	$0.8\overline{33}$
10	$0.\overline{40}$
100	0.04
1000	0.004

(b) The zeros of the denominator are $x = \pm 1$ so both $x = 1$ and $x = -1$ are vertical asymptotes. Because the degree of the numerator is less than the degree of the denominator, the x-axis or $y = 0$ is a horizontal asymptote.

(c) The domain is all real numbers except $x = \pm 1$.

5. $f(x) = \dfrac{1}{x^2}$

Domain: all real numbers except $x = 0$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

[Degree of $N(x)$ < degree of $D(x)$]

6. $f(x) = \dfrac{4}{(x-2)^3}$

Domain: all real numbers except $x = 2$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

[Degree of $N(x)$ < degree of $D(x)$]

7. $f(x) = \dfrac{2 + x}{2 - x} = \dfrac{x + 2}{-x + 2}$

Domain: all real numbers except $x = 2$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = -1$

[Degree of $N(x)$ = degree of $D(x)$]

8. $f(x) = \dfrac{1 - 5x}{1 + 2x} = \dfrac{-5x + 1}{2x + 1}$

Domain: all real numbers except $x = -\dfrac{1}{2}$

Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = -\dfrac{5}{2}$

[Degree of $N(x)$ = degree of $D(x)$]

9. $f(x) = \dfrac{x^3}{x^2 - 1}$

Domain: all real numbers except $x = \pm 1$

Vertical asymptotes: $x = \pm 1$

Horizontal asymptote: None

[Degree of $N(x)$ > degree of $D(x)$]

10. $f(x) = \dfrac{2x^2}{x + 1}$

Domain: all real numbers except $x = -1$

Vertical asymptote: $x = -1$

Horizontal asymptote: None

[Degree of $N(x)$ > degree of $D(x)$]

11. $f(x) = \dfrac{3x^2 + 1}{x^2 + x + 9}$

Domain: All real numbers. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]

Vertical asymptote: None

Horizontal asymptote: $y = 3$

[Degree of $N(x)$ = degree of $D(x)$]

12. $f(x) = \dfrac{3x^2 + x - 5}{x^2 + 1}$

Domain: All real numbers. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]

Vertical asymptote: None

Horizontal asymptote: $y = 3$

[Degree of $N(x)$ = degree of $D(x)$]

13. $f(x) = \dfrac{2}{x + 3}$

Vertical asymptote: $y = -3$

Horizontal asymptote: $y = 0$

Matches graph (d).

14. $f(x) = \dfrac{1}{x - 5}$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = 0$

Matches graph (a).

15. $f(x) = \dfrac{x - 1}{x - 4}$

Vertical asymptote: $x = 4$

Horizontal asymptote: $y = 1$

Matches graph (c).

16. $f(x) = -\dfrac{x + 2}{x + 4}$

Vertical asymptote: $x = -4$

Horizontal asymptote: $y = -1$

Matches graph (b).

17. $g(x) = \dfrac{x^2 - 1}{x + 1} = \dfrac{(x - 1)(x + 1)}{x + 1}$

The only zero of $g(x)$ is $x = 1$.

$x = -1$ makes $g(x)$ undefined.

18. $h(x) = 2 + \dfrac{5}{x^2 + 2}$

$$\begin{aligned} 0 &= 2 + \frac{5}{x^2 + 2} \\ -2 &= \frac{5}{x^2 + 2} \\ -2(x^2 + 2) &= 5 \\ x^2 &= -\frac{5}{2} - 2 \end{aligned}$$

No real solution, $h(x)$ has no real zeros.

19. $f(x) = 1 - \dfrac{3}{x - 3}$

$$1 - \frac{3}{x-3} = 0$$

$$1 = \frac{3}{x-3}$$

$$x - 3 = 3$$

$x = 6$ is a zero of $f(x)$.

20. $g(x) = \dfrac{x^3 - 8}{x^2 + 1}$

$$\frac{x^3 - 8}{x^2 + 1} = 0$$

$$x^3 - 8 = 0$$

$$x^3 = 8$$

$$x = 2$$

$x = 2$ is a real zero of $g(x)$.

21. $f(x) = \dfrac{x - 4}{x^2 - 16} = \dfrac{1}{x + 4},\ x \neq 4$

Domain: all real numbers x except $x = \pm 4$

Horizontal asymptote: $y = 0$

[Degree of $N(x)$ < degree of $D(x)$]

Vertical asymptote: $x = -4$ (Since $x - 4$ is a common factor of $N(x)$ and $D(x)$, $x = 4$ is not a vertical asymptote of $f(x)$.)

22. $f(x) = \dfrac{x + 3}{x^2 - 9} = \dfrac{x + 3}{(x + 3)(x - 3)} = \dfrac{1}{x - 3},\ x \neq -3$

Domain: all real numbers x except $x = \pm 3$

The degree of the numerator is less than the degree of the denominator, so the graph has the line $y = 0$ as a horizontal asymptote.

Vertical asymptote: $x = 3$ (Since $x + 3$ is a common factor of $N(x)$ and $D(x)$, $x = -3$ is not a vertical asymptote of $f(x)$.)

23. $f(x) = \dfrac{x^2 - 1}{x^2 - 2x - 3} = \dfrac{(x + 1)(x - 1)}{(x + 1)(x - 3)} = \dfrac{x - 1}{x - 3},\ x \neq -1$

Domain: all real numbers x except $x = -1$ and $x = 3$

Horizontal asymptote: $y = 1$

[Degree of $N(x)$ = degree of $D(x)$]

Vertical asymptote: $x = 3$
(Since $x + 1$ is a common factor of $N(x)$ and $D(x)$, $x = -1$ is not a vertical asymptote of $f(x)$.)

24. $f(x) = \dfrac{x^2 - 4}{x^2 - 3x + 2}$

$$= \frac{(x - 2)(x + 2)}{(x - 2)(x - 1)} = \frac{x + 2}{x - 1},\ x \neq 2$$

Domain: all real numbers x except $x = 1$ and $x = 2$

Horizontal asymptote: $y = 1$

[Degree of $N(x)$ = degree of $D(x)$]

Vertical asymptote: $x = 1$ (Since $x - 2$ is a common factor of $N(x)$ and $D(x)$, $x = 2$ is not a vertical asymptote of $f(x)$.)

25. $f(x) = \dfrac{x^2 - 3x - 4}{2x^2 + x - 1}$

$$= \frac{(x + 1)(x - 4)}{(2x - 1)(x + 1)} = \frac{x - 4}{2x - 1},\ x \neq -1$$

Domain: all real numbers x except $x = \dfrac{1}{2}$ and $x = -1$

Horizontal asymptote: $y = \dfrac{1}{2}$

[Degree of $N(x)$ = degree of $D(x)$]

Vertical asymptote: $x = \frac{1}{2}$ (Since $x + 1$ is a common factor of $N(x)$ and $D(x)$, $x = -1$ is not a vertical asymptote of $f(x)$.)

26. $f(x) = \dfrac{6x^2 - 11x + 3}{6x^2 - 7x - 3}$

$$= \frac{(2x - 3)(3x - 1)}{(2x - 3)(3x + 1)} = \frac{3x - 1}{3x + 1},\ x \neq \frac{3}{2}$$

Domain: all real numbers x except $x = \dfrac{3}{2}$ or $x = -\dfrac{1}{3}$

Horizontal asymptote: $y = 1$

[Degree of $N(x)$ = degree of $D(x)$]

Vertical asymptote: $x = -\frac{1}{3}$ (Since $2x - 3$ is a common factor of $N(x)$ and $D(x)$, $x = \frac{3}{2}$ is not a vertical asymptote of $f(x)$.)

27. $f(x) = \dfrac{1}{x + 2}$

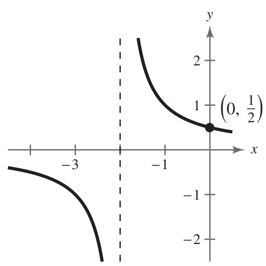
(a) Domain: all real numbers x except $x = -2$

(b) y-intercept: $\left(0, \dfrac{1}{2}\right)$

(c) Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 0$

(d)

x	-4	-3	-1	0	1
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$



28. $f(x) = \dfrac{1}{x - 3}$

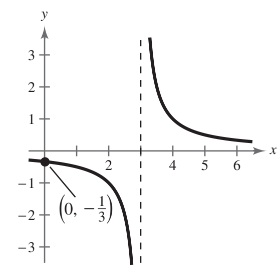
(a) Domain: all real numbers x except $x = 3$

(b) y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$

(d)

x	0	1	2	4	5	6
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$



29. $h(x) = \dfrac{-1}{x + 2}$

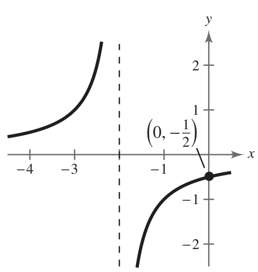
(a) Domain: all real numbers x except $x = -2$

(b) y-intercept: $\left(0, -\dfrac{1}{2}\right)$

(c) Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 0$

(d)

x	-4	-3	-1	0
y	$\frac{1}{2}$	1	-1	$-\frac{1}{2}$



Note: This is the graph of $f(x) = \dfrac{1}{x + 2}$ (Exercise 27) reflected about the x-axis.

30. $g(x) = \dfrac{1}{3 - x} = -\dfrac{1}{x - 3}$

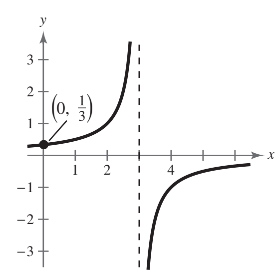
(a) Domain: all real numbers x except $x = 3$

(b) y-intercept: $\left(0, \dfrac{1}{3}\right)$

(c) Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$

(d)

x	0	1	2	4	5	6
y	$\frac{1}{3}$	$\frac{1}{2}$	1	-1	$-\frac{1}{2}$	$-\frac{1}{3}$



Note: This is the graph of $f(x) = \dfrac{1}{x - 3}$ (Exercise 28) reflected about the x-axis.

31. $C(x) = \dfrac{5 + 2x}{1 + x} = \dfrac{2x + 5}{x + 1}$

(a) Domain: all real numbers x except $x = -1$

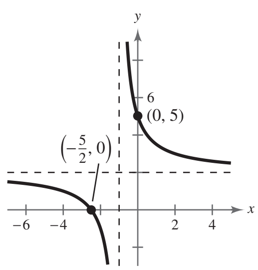
(b) x-intercept: $\left(-\dfrac{5}{2}, 0\right)$

y-intercept: $(0, 5)$

(c) Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 2$

(d)

x	-4	-3	-2	0	1	2
$C(x)$	1	$\frac{1}{2}$	-1	5	$\frac{7}{2}$	3



32. $P(x) = \dfrac{1 - 3x}{1 - x} = \dfrac{3x - 1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

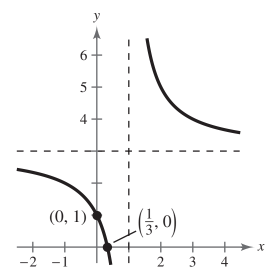
(b) x-intercept: $\left(\dfrac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 3$

(d)

x	-1	0	2	3
y	2	1	5	4



33. $f(x) = \dfrac{x^2}{x^2 + 9}$

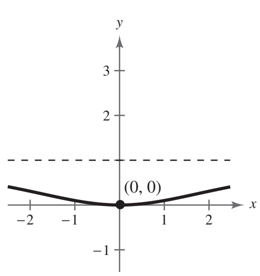
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 1$

(d)

x	± 1	± 2	± 3
y	$\frac{1}{10}$	$\frac{4}{13}$	$\frac{1}{2}$



34. $f(t) = \dfrac{1 - 2t}{t} = -\dfrac{2t - 1}{t}$

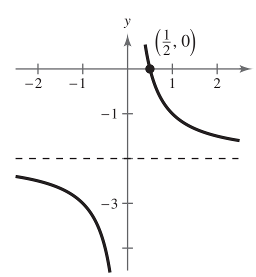
(a) Domain: all real numbers t except $t = 0$

(b) t-intercept: $\left(\dfrac{1}{2}, 0\right)$

(c) Vertical asymptote: $t = 0$
Horizontal asymptote: $y = -2$

(d)

t	-2	-1	$\frac{1}{2}$	1	2
y	$-\frac{5}{2}$	-3	0	-1	$-\frac{3}{2}$



35. $g(s) = \dfrac{s}{s^2 + 1}$

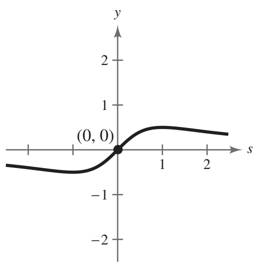
(a) Domain: all real numbers s

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

s	-2	-1	0	1	2
$g(s)$	$-\frac{2}{5}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{2}{5}$



36. $f(x) = -\dfrac{1}{(x-2)^2}$

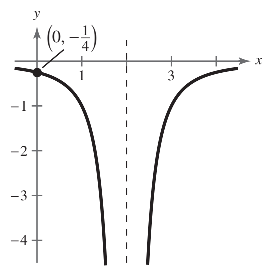
(a) Domain: all real numbers x except $x = 2$

(b) y-intercept: $\left(0, -\dfrac{1}{4}\right)$

(c) Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 0$

(d)

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}$	3	$\frac{7}{2}$	4
y	$-\frac{1}{4}$	$-\frac{4}{9}$	-1	-4	-4	-1	$-\frac{4}{9}$	$-\frac{1}{4}$



37. $h(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4} = \dfrac{(x-1)(x-4)}{(x+2)(x-2)}$

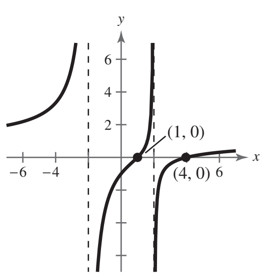
(a) Domain: all real numbers x except $x = \pm 2$

(b) x-intercepts: $(1, 0), (4, 0)$
y-intercept: $(0, -1)$

(c) Vertical asymptotes: $x = -2, x = 2$
Horizontal asymptote: $y = 1$

(d)

x	-4	-3	-1	0	1	3	4
y	$\frac{10}{3}$	$\frac{28}{5}$	$-\frac{10}{3}$	-1	0	$-\frac{2}{5}$	0



38. $g(x) = \dfrac{x^2 - 2x - 8}{x^2 - 9} = \dfrac{(x-4)(x+2)}{(x-3)(x+3)}$

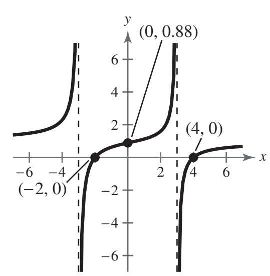
(a) Domain: all real numbers x except $x = \pm 3$

(b) y-intercept: $\left(0, \dfrac{8}{9}\right)$
x-intercepts: $(4, 0), (-2, 0)$

(c) Vertical asymptotes: $x = \pm 3$
Horizontal asymptote: $y = 1$

(d)

x	-5	-4	-2	0	2	4	5
y	$\frac{27}{16}$	$\frac{16}{7}$	0	$\frac{8}{9}$	$\frac{8}{5}$	0	$\frac{7}{16}$



39. $f(x) = \dfrac{2x^2 - 5x - 3}{x^3 - 2x^2 - x + 2} = \dfrac{(2x + 1)(x - 3)}{(x - 2)(x + 1)(x - 1)}$

(a) Domain: all real numbers x except $x = 2, x = \pm 1$

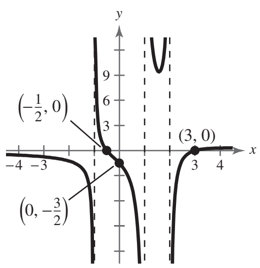
(b) x-intercepts: $\left(-\dfrac{1}{2}, 0\right), (3, 0)$

y-intercept: $\left(0, -\dfrac{3}{2}\right)$

(c) Vertical asymptotes: $x = 2, x = -1$ and $x = 1$
Horizontal asymptotes: $y = 0$

(d)

x	-3	-2	0	1.5	3	4
$f(x)$	$-\frac{3}{4}$	$-\frac{5}{4}$	$-\frac{3}{2}$	$\frac{48}{5}$	0	$\frac{3}{10}$



40. $f(x) = \dfrac{x^2 - x - 2}{x^3 - 2x^2 - 5x + 6} = \dfrac{(x + 1)(x - 2)}{(x - 1)(x + 2)(x - 3)}$

(a) Domain: all real numbers x except $x = 1, x = -2$, or $x = 3$

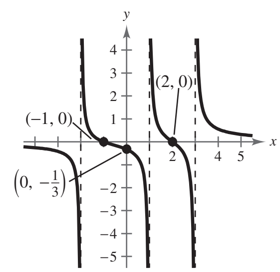
(b) x-intercepts: $(-1, 0), (2, 0)$

y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptotes: $x = -2, x = 1, x = 3$
Horizontal asymptote: $y = 0$

(d)

x	-4	-3	-1	0	2	4
y	$-\frac{9}{35}$	$-\frac{5}{12}$	0	$-\frac{1}{3}$	0	$\frac{5}{9}$



41. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x + 3)}{(x + 3)(x - 2)} = \dfrac{x}{x - 2}, \quad x \neq -3$

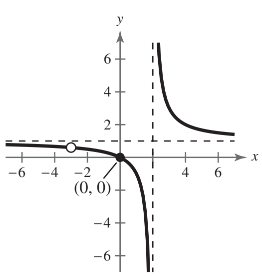
(a) Domain: all real numbers x except $x = -3$ and $x = 2$

(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 1$

(d)

x	-1	0	1	3	4
y	$\frac{1}{3}$	0	-1	3	2



42. $f(x) = \dfrac{5(x + 4)}{x^2 + x - 12} = \dfrac{5(x + 4)}{(x + 4)(x - 3)} = \dfrac{5}{x - 3}, \quad x \neq -4$

(a) Domain: all real numbers x except $x = -4$ or $x = 3$

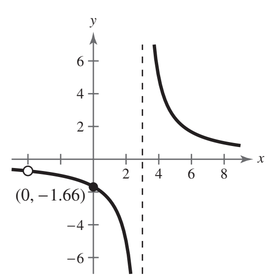
(b) y-intercept: $\left(0, -\dfrac{5}{3}\right)$

x-intercept: none

(c) Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$

(d)

x	-2	0	2	5	7
y	-1	$-\frac{5}{3}$	-5	$\frac{5}{2}$	$\frac{5}{4}$



43. $f(x) = \dfrac{2x^2 - 5x + 2}{2x^2 - x - 6}$

$= \dfrac{(2x - 1)(x - 2)}{(2x + 3)(x - 2)} = \dfrac{2x - 1}{2x + 3},\ \ x \neq 2$

(a) Domain: all real numbers x except $x = 2$ and $x = -\frac{3}{2}$

(b) x-intercept: $\left(\frac{1}{2}, 0\right)$

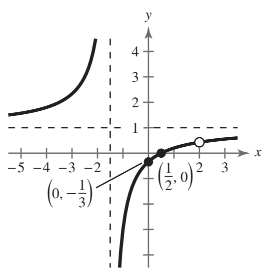
y-intercept: $\left(0, -\frac{1}{3}\right)$

(c) Vertical asymptote: $x = -\frac{3}{2}$

Horizontal asymptote: $y = 1$

(d)

x	-3	-2	-1	0	1
y	$\frac{7}{3}$	5	-3	$-\frac{1}{3}$	$\frac{1}{5}$



44. $f(x) = \dfrac{3x^2 - 8x + 4}{2x^2 - 3x - 2}$

$= \dfrac{(x - 2)(3x - 2)}{(x - 2)(2x + 1)} = \dfrac{3x - 2}{2x + 1},\ \ x \neq 2$

(a) Domain: all real numbers x except $x = 2$ or $x = -\frac{1}{2}$

(b) y-intercept: $(0, -2)$

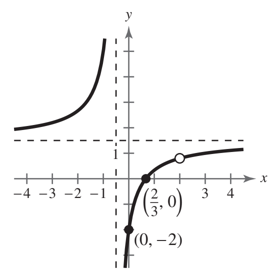
x-intercept: $\left(\frac{2}{3}, 0\right)$

(c) Vertical asymptote: $x = -\frac{1}{2}$

Horizontal asymptote: $y = \frac{3}{2}$

(d)

x	-3	-1	0	$\frac{2}{3}$	3
y	$\frac{11}{5}$	5	-2	0	1



45. $f(t) = \dfrac{t^2 - 1}{t + 1} = \dfrac{(t + 1)(t - 1)}{t + 1} = t - 1,\ \ t \neq -1$

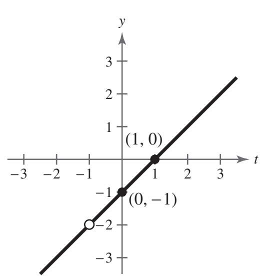
(a) Domain: all real numbers t except $t = -1$

(b) t-intercept: $(1, 0)$
y-intercept: $(0, -1)$

(c) Vertical asymptote: none
Horizontal asymptote: none

(d)

t	-3	-2	0	1
y	-4	-3	-1	0



46. $f(x) = \dfrac{x^2 - 16}{x - 4} = \dfrac{(x - 4)(x + 4)}{x - 4} = x + 4,\ \ x \neq 4$

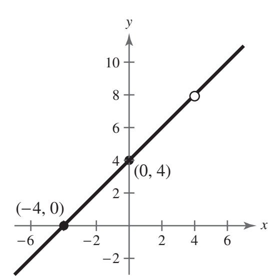
(a) Domain: all real numbers $x \neq 4$

(b) y-intercept: $(0, 4)$
x-intercept: $(-4, 0)$

(c) Vertical asymptote: none
Horizontal asymptote: none

(d)

x	-6	-4	0	5
y	-2	0	4	9



47. $f(x) = \dfrac{x^2 - 1}{x + 1}$, $g(x) = x - 1$

(a) Domain of f: all real numbers x except $x = -1$

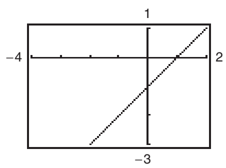
Domain of g: all real numbers x

(b) Because $(x + 1)$ is a factor of both the numerator and the denominator of f, $x = -1$ is not a vertical asymptote. f has no vertical asymptotes.

(c)

x	-3	-2	-1.5	-1	-0.5	0	1
$f(x)$	-4	-3	-2.5	Undef.	-1.5	-1	0
$g(x)$	-4	-3	-2.5	-2	-1.5	-1	0

(d)



(e) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

48. $f(x) = \dfrac{x^2(x - 2)}{x^2 - 2x}$, $g(x) = x$

(a) Domain of f: all real numbers x except 0 and 2

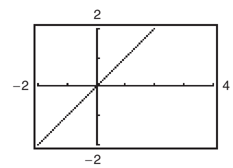
Domain of g: all real numbers x

(b) Since $x^2 - 2x$ is a factor of both the numerator and the denominator of f, neither $x = 0$ nor $x = 2$ is a vertical asymptote of f. Thus, f has no vertical asymptotes.

(c)

x	-1	0	1	1.5	2	2.5	3
$f(x)$	-1	Undef.	1	1.5	Undef.	2.5	3
$g(x)$	-1	0	1	1.5	2	2.5	3

(d)



(e) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

49. $f(x) = \dfrac{x - 2}{x^2 - 2x}$, $g(x) = \dfrac{1}{x}$

(a) Domain of f: all real numbers x except $x = 0$ and $x = 2$

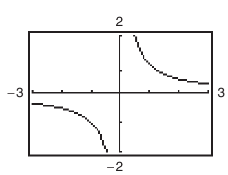
Domain of g: all real numbers x except $x = 0$

(b) Because $(x - 2)$ is a factor of both the numerator and the denominator of f, $x = 2$ is not a vertical asymptote. The only vertical asymptote of f is $x = 0$.

(c)

x	-0.5	0	0.5	1	1.5	2	3
$f(x)$	-2	Undef.	2	1	$\frac{2}{3}$	Undef.	$\frac{1}{3}$
$g(x)$	-2	Undef.	2	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$

(d)



(e) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

50. $f(x) = \dfrac{2x - 6}{x^2 - 7x + 12}$, $g(x) = \dfrac{2}{x - 4}$

(a) Domain of f: all real numbers x except 3 and 4

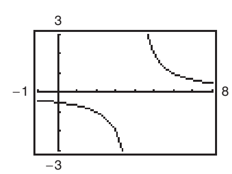
Domain of g: all real numbers x except 4

(b) Since $x - 3$ is a factor of both the numerator and the denominator of f, $x = 3$ is not a vertical asymptote of f. Thus, f has $x = 4$ as its only vertical asymptote.

(c)

x	0	1	2	3	4	5	6
$f(x)$	$-\frac{1}{2}$	$-\frac{2}{3}$	-1	Undef.	Undef.	2	1
$g(x)$	$-\frac{1}{2}$	$-\frac{2}{3}$	-1	-2	Undef.	2	1

(d)



(e) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

51. $h(x) = \dfrac{x^2 - 4}{x} = x - \dfrac{4}{x}$

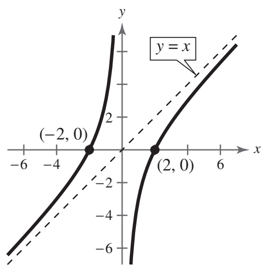
(a) Domain: all real numbers x except $x = 0$

(b) Intercepts: $(2, 0)$, $(-2, 0)$

(c) Vertical asymptote: $x = 0$
Slant asymptote: $y = x$

(d)

x	-3	-1	1	3
y	$-\frac{5}{3}$	3	-3	$\frac{5}{3}$



52. $g(x) = \dfrac{x^2 + 5}{x} = x + \dfrac{5}{x}$

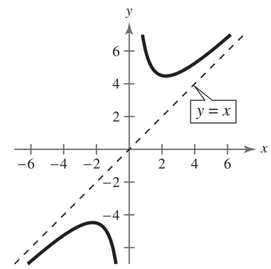
(a) Domain: all real numbers x except $x = 0$

(b) No intercepts

(c) Vertical asymptote: $x = 0$
Slant asymptote: $y = x$

(d)

x	-3	-2	-1	1	2	3
y	$-\frac{14}{3}$	$-\frac{9}{2}$	-6	6	$\frac{9}{2}$	$\frac{14}{3}$



53. $f(x) = \dfrac{2x^2 + 1}{x} = 2x + \dfrac{1}{x}$

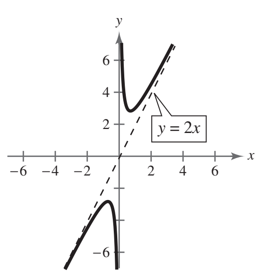
(a) Domain: all real numbers x except $x = 0$

(b) No intercepts

(c) Vertical asymptote: $x = 0$
Slant asymptote: $y = 2x$

(d)

x	-4	-2	2	4	6
$f(x)$	$-\frac{33}{4}$	$-\frac{9}{2}$	$\frac{9}{2}$	$\frac{33}{4}$	$\frac{73}{6}$



54. $f(x) = \dfrac{1 - x^2}{x} = -x + \dfrac{1}{x}$

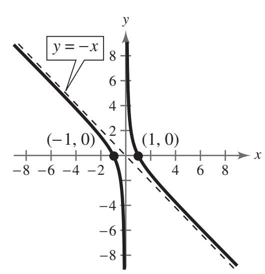
(a) Domain: all real numbers x except $x = 0$

(b) x-intercepts: $(-1, 0)$, $(1, 0)$

(c) Vertical asymptote: $x = 0$
Slant asymptote: $y = -x$

(d)

x	-6	-4	-2	2	4	6
$f(x)$	$\frac{35}{6}$	$\frac{15}{4}$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{15}{4}$	$-\frac{35}{6}$



55. $g(x) = \dfrac{x^2 + 1}{x} = x + \dfrac{1}{x}$

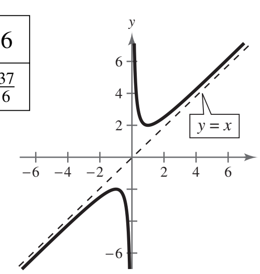
(a) Domain: all real numbers x except $x = 0$

(b) No intercepts

(c) Vertical asymptote: $x = 0$
Slant asymptote: $y = x$

(d)

x	-4	-2	2	4	6
$g(x)$	$-\frac{17}{4}$	$-\frac{5}{2}$	$\frac{5}{2}$	$\frac{17}{4}$	$\frac{37}{6}$



56. $h(x) = \dfrac{x^2}{x-1} = x + 1 + \dfrac{1}{x-1}$

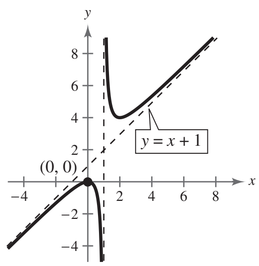
(a) Domain: all real numbers x except $x = 1$

(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = 1$
Slant asymptote: $y = x + 1$

(d)

x	-4	-2	2	4	6
$h(x)$	$-\frac{16}{5}$	$-\frac{4}{3}$	4	$\frac{16}{3}$	$\frac{36}{5}$



57. $f(t) = -\dfrac{t^2 + 1}{t + 5} = -t + 5 - \dfrac{26}{t + 5}$

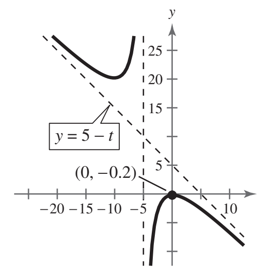
(a) Domain: all real numbers t except $t = -5$

(b) Intercept: $\left(0, -\dfrac{1}{5}\right)$

(c) Vertical asymptote: $t = -5$
Slant asymptote: $y = -t + 5$

(d)

t	-7	-6	-4	-3	0
y	25	37	-17	-5	$-\frac{1}{5}$



58. $f(x) = \dfrac{x^2}{3x + 1} = \dfrac{1}{3}x - \dfrac{1}{9} + \dfrac{1}{9(3x + 1)}$

(a) Domain: all real numbers x except $x = -\dfrac{1}{3}$

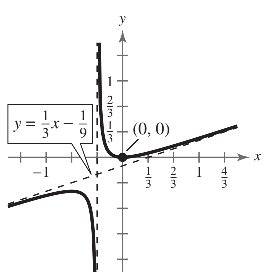
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = -\dfrac{1}{3}$

Slant asymptote: $y = \dfrac{1}{3}x - \dfrac{1}{9}$

(d)

x	-3	-2	-1	$-\frac{1}{2}$	0	2
y	$-\frac{9}{8}$	$-\frac{4}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{4}{7}$



59. $f(x) = \dfrac{x^3}{x^2 - 1} = x + \dfrac{x}{x^2 - 1}$

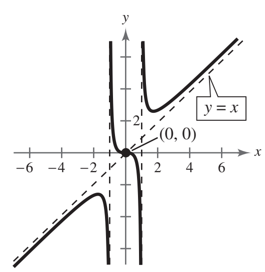
(a) Domain: all real numbers x except $x = \pm 1$

(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = \pm 1$
Slant asymptote: $y = x$

(d)

x	-4	-2	0	2	4
$f(x)$	$-\frac{64}{15}$	$-\frac{8}{3}$	0	$\frac{8}{3}$	$\frac{64}{15}$



60. $g(x) = \dfrac{x^3}{2x^2 - 8} = \dfrac{1}{2}x + \dfrac{4x}{2x^2 - 8}$

(a) Domain: all real numbers x except $x = \pm 2$

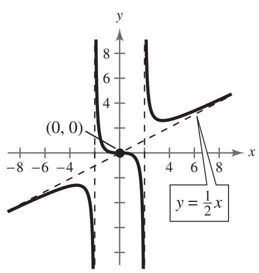
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = \dfrac{1}{2}x$

(d)

x	-6	-4	-1	1	4	6
$g(x)$	$-\frac{27}{8}$	$-\frac{8}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{8}{3}$	$\frac{27}{8}$



61. $f(x) = \dfrac{x^2 - x + 1}{x - 1} = x + \dfrac{1}{x - 1}$

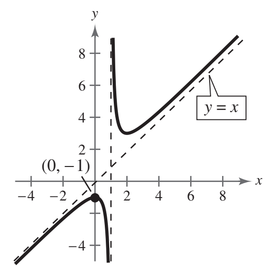
(a) Domain: all real numbers x except $x = 1$

(b) y-intercept: $(0, -1)$

(c) Vertical asymptote: $x = 1$
Slant asymptote: $y = x$

(d)

x	-4	-2	0	2	4
$f(x)$	$-\frac{21}{5}$	$-\frac{7}{3}$	-1	3	$\frac{13}{3}$



62. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \dfrac{3}{x - 2}$

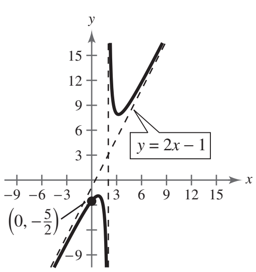
(a) Domain: all real numbers x except $x = 2$

(b) y-intercept: $\left(0, -\dfrac{5}{2}\right)$

(c) Vertical asymptote: $x = 2$
Slant asymptote: $y = 2x - 1$

(d)

x	-6	-3	1	3	6	7
y	$-\frac{107}{8}$	$-\frac{38}{5}$	-2	8	$\frac{47}{4}$	$\frac{68}{5}$



63. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

$$= \frac{(2x - 1)(x + 1)(x - 1)}{(x + 1)(x + 2)}$$

$$= \frac{(2x - 1)(x - 1)}{x + 2}, \quad x \neq -1$$

$$= \frac{2x^2 - 3x + 1}{x + 2}$$

$$= 2x - 7 + \frac{15}{x + 2}, \quad x \neq -1$$

(a) Domain: all real numbers x except $x = -1$ and $x = -2$

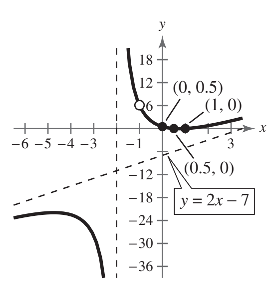
(b) y-intercept: $\left(0, \dfrac{1}{2}\right)$

x-intercepts: $\left(\dfrac{1}{2}, 0\right), (1, 0)$

(c) Vertical asymptote: $x = -2$
Slant asymptote: $y = 2x - 7$

(d)

x	-4	-3	$-\frac{3}{2}$	0	1
y	$-\frac{45}{2}$	-28	20	$\frac{1}{2}$	0



64. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2} = \dfrac{(x - 2)(x + 2)(2x + 1)}{(x - 2)(x - 1)}$

$= 2x + 7 + \dfrac{9}{x - 1}, x \neq 2$

(a) Domain: all real numbers x except $x = 1$ or $x = 2$

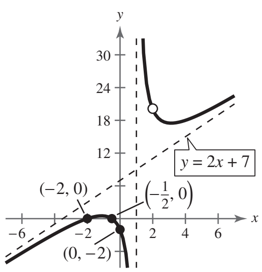
(b) y-intercept: $(0, -2)$

x-intercepts: $(-2, 0), \left(-\dfrac{1}{2}, 0\right)$

(c) Vertical asymptote: $x = 1$
Slant asymptote: $y = 2x + 7$

(d)

x	-3	-2	-1	0	$\frac{1}{2}$	$\frac{3}{2}$	3	4
y	$-\frac{5}{4}$	0	$\frac{1}{2}$	-2	-10	28	$\frac{35}{2}$	18



65. $f(x) = \dfrac{x^2 + 5x + 8}{x + 3} = x + 2 + \dfrac{2}{x + 3}$

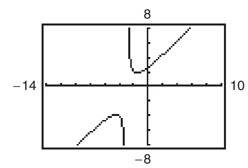
Domain: all real numbers x except $x = -3$

y-intercept: $\left(0, \dfrac{8}{3}\right)$

Vertical asymptote: $x = -3$

Slant asymptote: $y = x + 2$

Line: $y = x + 2$



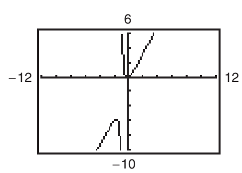
66. $f(x) = \dfrac{2x^2 + x}{x + 1} = 2x - 1 + \dfrac{1}{x + 1}$

Domain: all real numbers x except $x = -1$

Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x - 1$

Line: $y = 2x - 1$



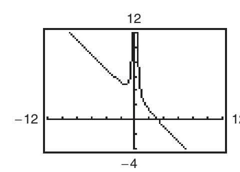
67. $g(x) = \dfrac{1 + 3x^2 - x^3}{x^2} = \dfrac{1}{x^2} + 3 - x = -x + 3 + \dfrac{1}{x^2}$

Domain: all real numbers x except $x = 0$

Vertical asymptote: $x = 0$

Slant asymptote: $y = -x + 3$

Line: $y = -x + 3$



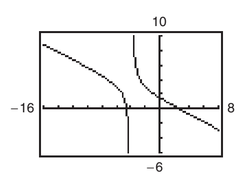
68. $h(x) = \dfrac{12 - 2x - x^2}{2(4 + x)} = -\dfrac{1}{2}x + 1 + \dfrac{2}{4 + x}$

Domain: all real numbers x except $x = -4$

Vertical asymptote: $x = -4$

Slant asymptote: $y = -\dfrac{1}{2}x + 1$

Line: $y = -\dfrac{1}{2}x + 1$



69. $y = \dfrac{x + 1}{x - 3}$

(a) x-intercept: $(-1, 0)$

(b) $0 = \dfrac{x + 1}{x - 3}$

$0 = x + 1$

$-1 = x$

70. (a) x-intercept: $(0, 0)$

(b) $0 = \dfrac{2x}{x - 3}$

$0 = 2x$

$0 = x$

71. $y = \dfrac{1}{x} - x$

(a) x-intercepts: $(\pm 1, 0)$

(b) $0 = \dfrac{1}{x} - x$

$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$

72. (a) x-intercepts: $(1, 0), (2, 0)$

(b) $0 = x - 3 + \dfrac{2}{x}$

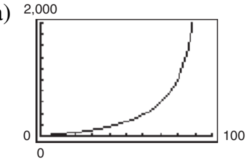
$0 = x^2 - 3x + 2$

$0 = (x - 1)(x - 2)$

$x = 1, x = 2$

73. $C = \dfrac{255p}{100 - p}, \; 0 \leq p < 100$

(a)



(b) $C(10) = \dfrac{255(10)}{100 - 10} \approx 28.33$ million dollars

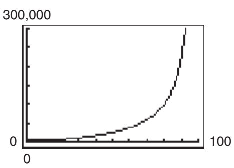
$C(40) = \dfrac{255(40)}{100 - 40} = 170$ million dollars

$C(75) = \dfrac{255(75)}{100 - 75} = 765$ million dollars

(c) $C \longrightarrow \infty$ as $x \longrightarrow 100$. No, it would not be possible to remove 100% of the pollutants.

74. $C = \dfrac{25{,}000p}{100 - p}, \; 0 \leq p < 100$

(a)



(b) $C = \dfrac{25{,}000(15)}{100 - 15} \approx 4411.76$

The cost would be \$4411.76.

$C = \dfrac{25{,}000(50)}{100 - 50} = 25{,}000$

The cost would be \$25,000.

$C = \dfrac{25{,}000(90)}{100 - 90} = 225{,}000$

The cost would be \$225,000.

(c) $C \longrightarrow \infty$ as $x \longrightarrow 100$. No. The model is undefined for $p = 100$.

75. $N = \dfrac{20(5 + 3t)}{1 + 0.04t}, \; t \geq 0$

(a) $N(5) \approx 333$ deer

$N(10) = 500$ deer

$N(25) = 800$ deer

(b) The herd is limited by the horizontal asymptote:

$N = \dfrac{60}{0.04} = 1500$ deer

76. (a) $0.25(50) + 0.75(x) = C(50 + x)$

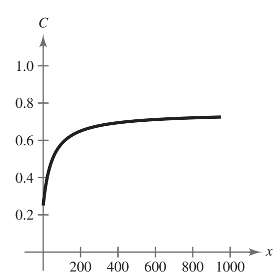
$C = \dfrac{12.50 + 0.75x}{50 + x} \cdot \dfrac{4}{4}$

$C = \dfrac{50 + 3x}{4(50 + x)} = \dfrac{3x + 50}{4(x + 50)}$

(b) Domain: $x \geq 0$ and $x \leq 1000 - 50$

Thus, $0 \leq x \leq 950$. Using interval notation, the domain is $[0, 950]$.

(c)



(d) As the tank is filled, the concentration increases more slowly. It approaches the horizontal asymptote of $C = \frac{3}{4} = 0.75$.

77. (a) $A = xy$ and

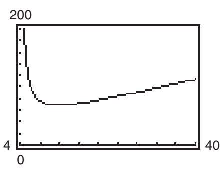
$$(x - 4)(y - 2) = 30$$

$$y - 2 = \frac{30}{x - 4}$$

$$y = 2 + \frac{30}{x - 4} = \frac{2x + 22}{x - 4}$$

Thus, $A = xy = x\left(\frac{2x + 22}{x - 4}\right) = \frac{2x(x + 11)}{x - 4}$.

(b) Domain: Since the margins on the left and right are each 2 inches, $x > 4$. In interval notation, the domain is $(4, \infty)$.

(c) 

x	5	6	7	8	9	10	11	12	13	14	15
y_1 (Area)	160	102	84	76	72	70	69.143	69	69.333	70	70.909

The area is minimum when $x \approx 11.75$ inches and $y \approx 5.87$ inches.

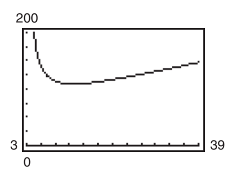
The area is minimum when x is approximately 12.

78. $A = xy$ and

$$(x - 3)(y - 2) = 64$$

$$y - 2 = \frac{64}{x - 3}$$

$$y = 2 + \frac{64}{x - 3} = \frac{2x + 58}{x - 3}$$



Thus, $A = xy = x\left(\frac{2x + 58}{x - 3}\right) = \frac{2x(x + 29)}{x - 3},\ x > 3$.

By graphing the area function, we see that A is minimum when $x \approx 12.8$ inches and $y \approx 8.5$ inches.

79. (a) Let t_1 = time from Akron to Columbus and t_2 = time from Columbus back to Akron.

$$xt_1 = 100 \implies t_1 = \frac{100}{x}$$

$$yt_2 = 100 \implies t_2 = \frac{100}{y}$$

$$50(t_1 + t_2) = 200$$

$$t_1 + t_2 = 4$$

$$\frac{100}{x} + \frac{100}{y} = 4$$

$$100y + 100x = 4xy$$

$$25y + 25x = xy$$

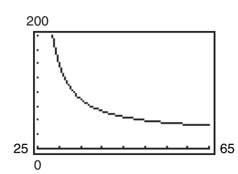
$$25x = xy - 25y$$

$$25x = y(x - 25)$$

Thus, $y = \frac{25x}{x - 25}$.

(b) Vertical asymptote: $x = 25$

Horizontal asymptote: $y = 25$

(c) 

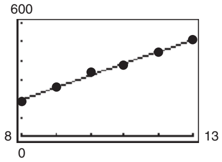
(d)

x	30	35	40	45	50	55	60
y	150	87.5	66.67	56.25	50	45.83	42.86

(e) Yes. You would expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

(f) No. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

80. (a)



(b) $S = \dfrac{5.816(18)^2 - 130.68}{0.004(18)^2 + 1.00} = 763.81$

The sales in 2008 is estimated to be $763,810,000.

(c) Probably not. The graph has a horizontal asymptote at $S = \dfrac{5.816}{0.004} \approx 1454$ million dollars.

Future sales may exceed this limiting value.

81. False. Polynomial functions do not have vertical asymptotes.

82. False. The graph of $f(x) = \dfrac{x}{x^2 + 1}$ crosses $y = 0$, which is a horizontal asymptote.

83. Vertical asymptote: None $\Longrightarrow$ The denominator is not zero for any value of x (unless the numerator is also zero there).

Horizontal asymptote: $y = 2 \Longrightarrow$ The degree of the numerator equals the degree of the denominator.

$f(x) = \dfrac{2x^2}{x^2 + 1}$ is one possible function. There are many correct answers.

84. Vertical asymptotes: $x = -2,\ x = 1 \Longrightarrow (x + 2)(x - 1)$ are factors of the denominator.

Horizontal asymptotes: None $\Longrightarrow$ The degree of the numerator is greater than the degree of the denominator.

$f(x) = \dfrac{x^3}{(x + 2)(x - 1)}$ is one possible function. There are many correct answers.

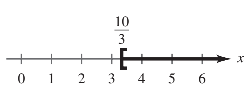
85. $x^2 - 15x + 56 = (x - 8)(x - 7)$

86. $3x^2 + 23x - 36 = (3x - 4)(x + 9)$

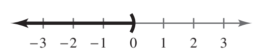
87.
$$\begin{aligned} x^3 - 5x^2 + 4x - 20 &= (x - 5)(x^2 + 4) \\ &= (x - 5)(x + 2i)(x - 2i) \end{aligned}$$

88.
$$\begin{aligned} x^3 + 6x^2 - 2x - 12 &= x^2(x + 6) - 2(x + 6) \\ &= (x + 6)(x^2 - 2) \\ &= (x + 6)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right) \end{aligned}$$

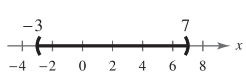
89.
$$\begin{aligned} 10 - 3x &\le 0 \\ 3x &\ge 10 \\ x &\ge \tfrac{10}{3} \end{aligned}$$



90.
$$\begin{aligned} 5 - 2x &> 5(x + 1) \\ 5 - 2x &> 5x + 5 \\ -7x &> 0 \\ x &< 0 \end{aligned}$$

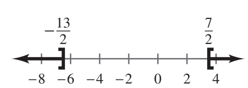


91.
$$\begin{aligned} |4(x - 2)| &< 20 \\ -20 < 4x - 8 &< 20 \\ -12 < 4x &< 28 \\ -3 < x &< 7 \end{aligned}$$



92.
$$\begin{aligned} \tfrac{1}{2}|2x + 3| &\ge 5 \\ |2x + 3| &\ge 10 \end{aligned}$$

$$\begin{aligned} 2x + 3 &\le -10 \quad \text{or} \quad & 2x + 3 &\ge 10 \\ 2x &\le -13 & 2x &\ge 7 \\ x &\le -\tfrac{13}{2} & x &\ge \tfrac{7}{2} \end{aligned}$$



93. Answers will vary.

Section 2.7 Nonlinear Inequalities

- You should be able to solve inequalities.
  - (a) Find the critical number.
    1. Values that make the expression zero
    2. Values that make the expression undefined
  - (b) Test one value in each test interval on the real number line resulting from the critical numbers.
  - (c) Determine the solution intervals.

Vocabulary Check

1. critical; test intervals **2.** zeros; undefined values **3.** $P = R - C$

1. $x^2 - 3 < 0$

(a) $x = 3$

$(3)^2 - 3 \overset{?}{<} 0$

$6 \not< 0$

No, $x = 3$ *is not* a solution.

(b) $x = 0$

$(0)^2 - 3 \overset{?}{<} 0$

$-3 < 0$

Yes, $x = 0$ *is* a solution.

(c) $x = \frac{3}{2}$

$\left(\frac{3}{2}\right)^2 - 3 \overset{?}{<} 0$

$-\frac{3}{4} < 0$

Yes, $x = \frac{3}{2}$ *is* a solution.

(d) $x = -5$

$(-5)^2 - 3 \overset{?}{<} 0$

$22 \not< 0$

No, $x = -5$ *is not* a solution.

2. $x^2 - x - 12 \geq 0$

(a) $x = 5$

$(5)^2 - (5) - 12 \overset{?}{\geq} 0$

$8 \geq 0$

Yes, $x = 5$ *is* a solution.

(b) $x = 0$

$(0)^2 - 0 - 12 \overset{?}{\geq} 0$

$-12 \not\geq 0$

No, $x = 0$ *is not* a solution.

(c) $x = -4$

$(-4)^2 - (-4) - 12 \overset{?}{\geq} 0$

$16 + 4 - 12 \overset{?}{\geq} 0$

$8 \geq 0$

Yes, $x = -4$ *is* a solution.

(d) $x = -3$

$(-3)^2 - (-3) - 12 \overset{?}{\geq} 0$

$9 + 3 - 12 \overset{?}{\geq} 0$

$0 \geq 0$

Yes, $x = -3$ *is* a solution.

3. $\dfrac{x + 2}{x - 4} \geq 3$

(a) $x = 5$

$\dfrac{5 + 2}{5 - 4} \overset{?}{\geq} 3$

$7 \geq 3$

Yes, $x = 5$ *is* a solution.

(b) $x = 4$

$\dfrac{4 + 2}{4 - 4} \overset{?}{\geq} 3$

$\dfrac{6}{0}$ is undefined.

No, $x = 4$ *is not* a solution.

(c) $x = -\dfrac{9}{2}$

$\dfrac{-\frac{9}{2} + 2}{-\frac{9}{2} - 4} \overset{?}{\geq} 3$

$\dfrac{5}{17} \not\geq 3$

No, $x = -\frac{9}{2}$ *is not* a solution.

(d) $x = \dfrac{9}{2}$

$\dfrac{\frac{9}{2} + 2}{\frac{9}{2} - 4} \overset{?}{\geq} 3$

$13 \geq 3$

Yes, $x = \frac{9}{2}$ *is* a solution.

4. $\dfrac{3x^2}{x^2+4} < 1$

(a) $x = -2$

$$\frac{3(-2)^2}{(-2)^2+4} \overset{?}{<} 1$$

$$\frac{12}{8} \not< 1$$

No, $x = -2$ *is not* a solution.

(b) $x = -1$

$$\frac{3(-1)^2}{(-1)^2+4} \overset{?}{<} 1$$

$$\frac{3}{5} < 1$$

Yes, $x = -1$ *is* a solution.

(c) $x = 0$

$$\frac{3(0)^2}{(0)^2+4} \overset{?}{<} 1$$

$$0 < 1$$

Yes, $x = 0$ *is* a solution.

(d) $x = 3$

$$\frac{3(3)^2}{(3)^2+4} \overset{?}{<} 1$$

$$\frac{27}{13} \not< 1$$

No, $x = 3$ *is not* a solution.

5. $2x^2 - x - 6 = (2x+3)(x-2)$

$2x + 3 = 0 \implies x = -\dfrac{3}{2}$

$x - 2 = 0 \implies x = 2$

Critical numbers: $x = -\dfrac{3}{2}, x = 2$

6. $9x^3 - 25x^2 = 0$

$x^2(9x - 25) = 0$

$x^2 = 0 \implies x = 0$

$9x - 25 = 0 \implies x = \dfrac{25}{9}$

The critical numbers are 0 and $\dfrac{25}{9}$.

7. $2 + \dfrac{3}{x-5} = \dfrac{2(x-5)+3}{x-5}$

$= \dfrac{2x-7}{x-5}$

$2x - 7 = 0 \implies x = \dfrac{7}{2}$

$x - 5 = 0 \implies x = 5$

Critical numbers: $x = \dfrac{7}{2}, x = 5$

8. $\dfrac{x}{x+2} - \dfrac{2}{x-1} = \dfrac{x(x-1) - 2(x+2)}{(x+2)(x-1)}$

$= \dfrac{x^2 - x - 2x - 4}{(x+2)(x-1)}$

$= \dfrac{(x-4)(x+1)}{(x+2)(x-1)}$

$(x-4)(x+1) = 0$

$x - 4 = 0 \implies x = 4$

$x + 1 = 0 \implies x = -1$

$(x+2)(x-1) = 0$

$x + 2 = 0 \implies x = -2$

$x - 1 = 0 \implies x = 1$

The critical numbers are $-2, -1, 1, 4$.

9. $x^2 \le 9$

$x^2 - 9 \le 0$

$(x+3)(x-3) \le 0$

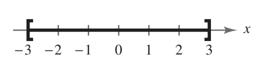
Critical numbers: $x = \pm 3$

Test intervals: $(-\infty, -3), (-3, 3), (3, \infty)$

Test: Is $(x+3)(x-3) \le 0$?

Interval	x-Value	Value of $x^2 - 9$	Conclusion
$(-\infty, -3)$	$x = -4$	$16 - 9 = 7$	Positive
$(-3, 3)$	$x = 0$	$0 - 9 = -9$	Negative
$(3, \infty)$	$x = 4$	$16 - 9 = 7$	Positive

Solution set: $[-3, 3]$



10. $x^2 < 36$

$x^2 - 36 < 0$

$(x+6)(x-6) < 0$

Critical numbers: $x = -6, x = 6$

Test intervals: $(-\infty, -6) \implies (x+6)(x-6) > 0$

$(-6, 6) \implies (x+6)(x-6) < 0$

$(6, \infty) \implies (x+6)(x-6) > 0$

Solution interval: $(-6, 6)$

11.
$$(x + 2)^2 < 25$$
$$x^2 + 4x + 4 < 25$$
$$x^2 + 4x - 21 < 0$$
$$(x + 7)(x - 3) < 0$$

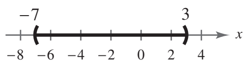
Critical numbers: $x = -7, x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

Test: Is $(x + 7)(x - 3) < 0$?

Interval	x-Value	Value of $(x + 7)(x - 3)$	Conclusion
$(-\infty, -7)$	$x = -10$	$(-3)(-13) = 39$	Positive
$(-7, 3)$	$x = 0$	$(7)(-3) = -21$	Negative
$(3, \infty)$	$x = 5$	$(12)(2) = 24$	Positive

Solution set: $(-7, 3)$



12.
$$(x - 3)^2 \ge 1$$
$$x^2 - 6x + 8 \ge 0$$
$$(x - 2)(x - 4) \ge 0$$

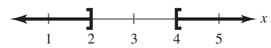
Critical numbers: $x = 2, x = 4$

Test intervals: $(-\infty, 2) \Rightarrow (x - 2)(x - 4) > 0$

$(2, 4) \Rightarrow (x - 2)(x - 4) < 0$

$(4, \infty) \Rightarrow (x - 2)(x - 4) > 0$

Solution intervals: $(-\infty, 2] \cup [4, \infty)$



13.
$$x^2 + 4x + 4 \ge 9$$
$$x^2 + 4x - 5 \ge 0$$
$$(x + 5)(x - 1) \ge 0$$

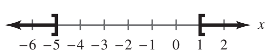
Critical numbers: $x = -5, x = 1$

Test intervals: $(-\infty, -5), (-5, 1), (1, \infty)$

Test: Is $(x + 5)(x - 1) \ge 0$?

Interval	x-Value	Value of $(x + 5)(x - 1)$	Conclusion
$(-\infty, -5)$	$x = -6$	$(-1)(-7) = 7$	Positive
$(-5, 1)$	$x = 0$	$(5)(-1) = -5$	Negative
$(1, \infty)$	$x = 2$	$(7)(1) = 7$	Positive

Solution set: $(-\infty, -5] \cup [1, \infty)$



14.
$$x^2 - 6x + 9 < 16$$
$$x^2 - 6x - 7 < 0$$
$$(x + 1)(x - 7) < 0$$

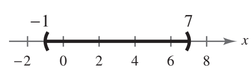
Critical numbers: $x = -1, x = 7$

Test intervals: $(-\infty, -1) \Rightarrow (x + 1)(x - 7) > 0$

$(-1, 7) \Rightarrow (x + 1)(x - 7) < 0$

$(7, \infty) \Rightarrow (x + 1)(x - 7) > 0$

Solution interval: $(-1, 7)$



15.
$$x^2 + x < 6$$
$$x^2 + x - 6 < 0$$
$$(x + 3)(x - 2) < 0$$

Critical numbers: $x = -3, x = 2$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Test: Is $(x + 3)(x - 2) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 2)$	Conclusion
$(-\infty, -3)$	$x = -4$	$(-1)(-6) = 6$	Positive
$(-3, 2)$	$x = 0$	$(3)(-2) = -6$	Negative
$(2, \infty)$	$x = 3$	$(6)(1) = 6$	Positive

Solution set: $(-3, 2)$

16.
$$x^2 + 2x > 3$$
$$x^2 + 2x - 3 > 0$$
$$(x + 3)(x - 1) > 0$$

Critical numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 1) > 0$

$(-3, 1) \Rightarrow (x + 3)(x - 1) < 0$

$(1, \infty) \Rightarrow (x + 3)(x - 1) > 0$

Solution intervals: $(-\infty, -3) \cup (1, \infty)$

17. $x^2 + 2x - 3 < 0$

$(x + 3)(x - 1) < 0$

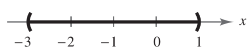
Critical numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3), (-3, 1), (1, \infty)$

Test: Is $(x + 3)(x - 1) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 1)$	Conclusion
$(-\infty, -3)$	$x = -4$	$(-1)(-5) = 5$	Positive
$(-3, 1)$	$x = 0$	$(3)(-1) = -3$	Negative
$(1, \infty)$	$x = 2$	$(5)(1) = 5$	Positive

Solution set: $(-3, 1)$



18. $x^2 - 4x - 1 > 0$

$x = \dfrac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5}$

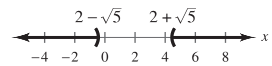
Critical numbers: $x = 2 - \sqrt{5}, x = 2 + \sqrt{5}$

Test intervals: $(-\infty, 2 - \sqrt{5}) \Rightarrow x^2 - 4x - 1 > 0$

$(2 - \sqrt{5}, 2 + \sqrt{5}) \Rightarrow x^2 - 4x - 1 < 0$

$(2 + \sqrt{5}, \infty) \Rightarrow x^2 - 4x - 1 > 0$

Solution intervals: $(-\infty, 2 - \sqrt{5}) \cup (2 + \sqrt{5}, \infty)$



19. $x^2 + 8x - 5 \geq 0$

$x^2 + 8x - 5 = 0$ Complete the square.

$x^2 + 8x + 16 = 5 + 16$

$(x + 4)^2 = 21$

$x + 4 = \pm\sqrt{21}$

$x = -4 \pm \sqrt{21}$

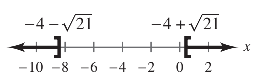
Critical numbers: $x = -4 \pm \sqrt{21}$

Test intervals: $(-\infty, -4 - \sqrt{21}), (-4 - \sqrt{21}, -4 + \sqrt{21}), (-4 + \sqrt{21}, \infty)$

Test: Is $x^2 + 8x - 5 \geq 0$?

Interval	x-Value	Value of $x^2 + 8x - 5$	Conclusion
$(-\infty, -4 - \sqrt{21})$	$x = -10$	$100 - 80 - 5 = 15$	Positive
$(-4 - \sqrt{21}, -4 + \sqrt{21})$	$x = 0$	$0 + 0 - 5 = -5$	Negative
$(-4 + \sqrt{21}, \infty)$	$x = 2$	$4 + 16 - 5 = 15$	Positive

Solution set: $(-\infty < -4 - \sqrt{21}] \cup [-4 + \sqrt{21}, \infty)$



20. $-2x^2 + 6x + 15 \leq 0$

$2x^2 - 6x - 15 \geq 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)} = \frac{6 \pm \sqrt{156}}{4} = \frac{6 \pm 2\sqrt{39}}{4} = \frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

Critical numbers: $x = \dfrac{3}{2} - \dfrac{\sqrt{39}}{2}, x = \dfrac{3}{2} + \dfrac{\sqrt{39}}{2}$

Test intervals: $\left(-\infty, \dfrac{3}{2} - \dfrac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 < 0$

$\left(\dfrac{3}{2} - \dfrac{\sqrt{39}}{2}, \dfrac{3}{2} + \dfrac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 > 0$

$\left(\dfrac{3}{2} + \dfrac{\sqrt{39}}{2}, \infty\right) \Rightarrow -2x^2 + 6x + 15 < 0$

Solution interval: $\left(-\infty, \dfrac{3}{2} - \dfrac{\sqrt{39}}{2}\right] \cup \left[\dfrac{3}{2} + \dfrac{\sqrt{39}}{2}, \infty\right)$

21. $x^3 - 3x^2 - x + 3 > 0$

$x^2(x - 3) - 1(x - 3) > 0$

$(x^2 - 1)(x - 3) > 0$

$(x + 1)(x - 1)(x - 3) > 0$

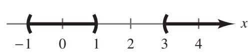
Critical numbers: $x = \pm 1, x = 3$

Test intervals: $(-\infty, -1), (-1, 1), (1, 3), (3, \infty)$

Test: Is $(x + 1)(x - 1)(x - 3) > 0$?

Interval	x-Value	Value of $(x + 1)(x - 1)(x - 3)$	Conclusion
$(-\infty, -1)$	$x = -2$	$(-1)(-3)(-5) = -15$	Negative
$(-1, 1)$	$x = 0$	$(1)(-1)(-3) = 3$	Positive
$(1, 3)$	$x = 2$	$(3)(1)(-1) = -3$	Negative
$(3, \infty)$	$x = 4$	$(5)(3)(1) = 15$	Positive

Solution set: $(-1, 1) \cup (3, \infty)$



22. $x^3 + 2x^2 - 4x - 8 \le 0$

$x^2(x + 2) - 4(x + 2) \le 0$

$(x + 2)(x^2 - 4) \le 0$

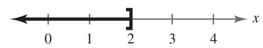
Critical numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \implies x^3 + 2x^2 - 4x - 8 < 0$

$(-2, 2) \implies x^3 + 2x^2 - 4x - 8 < 0$

$(2, \infty) \implies x^3 + 2x^2 - 4x - 8 > 0$

Solution interval: $(-\infty, 2]$



23. $x^3 - 2x^2 - 9x - 2 \ge -20$

$x^3 - 2x^2 - 9x + 18 \ge 0$

$x^2(x - 2) - 9(x - 2) \ge 0$

$(x - 2)(x^2 - 9) \ge 0$

$(x - 2)(x + 3)(x - 3) \ge 0$

Critical numbers: $x = 2, x = \pm 3$

Test intervals: $(-\infty, -3), (-3, 2), (2, 3), (3, \infty)$

Test: Is $(x - 2)(x + 3)(x - 3) \ge 0$?

Interval	x-Value	Value of $(x - 2)(x + 3)(x - 3)$	Conclusion
$(-\infty, -3)$	$x = -4$	$(-6)(-1)(-7) = -42$	Negative
$(-3, 2)$	$x = 0$	$(-2)(3)(-3) = 18$	Positive
$(2, 3)$	$x = 2.5$	$(0.5)(5.5)(-0.5) = -1.375$	Negative
$(3, \infty)$	$x = 4$	$(2)(7)(1) = 14$	Positive

Solution set: $[-3, 2] \cup [3, \infty)$

24. $2x^3 + 13x^2 - 8x - 46 \ge 6$

$2x^3 + 13x^2 - 8x - 52 \ge 0$

$x^2(2x + 13) - 4(2x + 13) \ge 0$

$(2x + 13)(x^2 - 4) \ge 0$

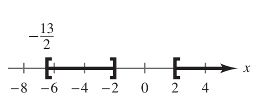
Critical numbers: $x = -\frac{13}{2}, x = -2, x = 2$

Test intervals: $\left(-\infty, -\frac{13}{2}\right) \implies 2x^3 + 13x^2 - 8x - 52 < 0$

$\left(-\frac{13}{2}, -2\right) \implies 2x^3 + 13x^2 - 8x - 52 > 0$

$(-2, 2) \implies 2x^3 + 13x^2 - 8x - 52 < 0$

$(2, \infty) \implies 2x^3 + 13x^2 - 8x - 52 > 0$



Solution interval: $\left[-\frac{13}{2}, -2\right], [2, \infty)$

25. $4x^2 - 4x + 1 \le 0$

$(2x - 1)^2 \le 0$

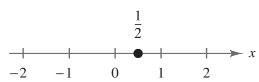
Critical number: $x = \frac{1}{2}$

Test intervals: $\left(-\infty, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

Test: Is $(2x - 1)^2 \le 0$?

Interval	x-Value	Value of $(2x - 1)^2$	Conclusion
$\left(-\infty, \frac{1}{2}\right)$	$x = 0$	$(-1)^2 = 1$	Positive
$\left(\frac{1}{2}, \infty\right)$	$x = 1$	$(1)^2 = 1$	Positive

Solution set: $x = \frac{1}{2}$

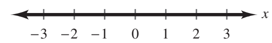


26. $x^2 + 3x + 8 > 0$

The critical numbers are imaginary:

$-\frac{3}{2} \pm \frac{i\sqrt{23}}{2}$

So the set of real numbers is the solution set.



27. $4x^3 - 6x^2 < 0$

$2x^2(2x - 3) < 0$

Critical numbers: $x = 0, x = \frac{3}{2}$

Test intervals: $(-\infty, 0), \left(0, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$

Test: Is $2x^2(2x - 3) < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(-\infty, 0) \cup \left(0, \frac{3}{2}\right)$

28. $4x^3 - 12x^2 > 0$

$4x^2(x - 3) > 0$

Critical numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \implies 4x^2(x - 3) < 0$

$(0, 3) \implies 4x^2(x - 3) < 0$

$(3, \infty) \implies 4x^2(x - 3) > 0$

Solution interval: $(3, \infty)$

29. $x^3 - 4x \ge 0$

$x(x + 2)(x - 2) \ge 0$

Critical numbers: $x = 0, x = \pm 2$

Test intervals: $(-\infty, -2), (-2, 0), (0, 2), (2, \infty)$

Test: Is $x(x + 2)(x - 2) \ge 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $[-2, 0] \cup [2, \infty)$

30. $2x^3 - x^4 \le 0$

$x^3(2 - x) \le 0$

Critical numbers: $x = 0, x = 2$

Test intervals: $(-\infty, 0) \implies x^3(2 - x) < 0$

$(0, 2) \implies x^3(2 - x) > 0$

$(2, \infty) \implies x^3(2 - x) < 0$

Solution intervals: $(-\infty, 0] \cup [2, \infty)$

31. $(x - 1)^2(x + 2)^3 \ge 0$

Critical numbers: $x = 1, x = -2$

Test intervals: $(-\infty, -2), (-2, 1), (1, \infty)$

Test: Is $(x - 1)^2(x + 3)^3 \ge 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $[-2, \infty)$

32. $x^4(x - 3) \le 0$

Critical numbers: $x = 0, x = 3$

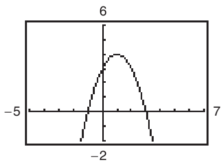
Test intervals: $(-\infty, 0) \implies x^4(x - 3) < 0$

$(0, 3) \implies x^4(x - 3) < 0$

$(3, \infty) \implies x^4(x - 3) > 0$

Solution intervals: $(-\infty, 0] \cup [0, 3]$ or $(-\infty, 3]$

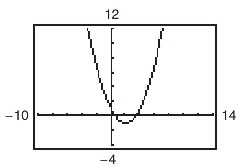
33. $y = -x^2 + 2x + 3$



(a) $y \le 0$ when $x \le -1$ or $x \ge 3$.

(b) $y \ge 3$ when $0 \le x \le 2$.

34. $y = \frac{1}{2}x^2 - 2x + 1$



(a) $y \le 0$

$$\frac{1}{2}x^2 - 2x + 1 \le 0$$

$$x^2 - 4x + 2 \le 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$$

$y \le 0$ when $2 - \sqrt{2} \le x \le 2 + \sqrt{2}$.

(b) $y \ge 7$

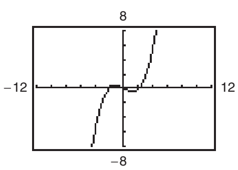
$$\frac{1}{2}x^2 - 2x + 1 \ge 7$$

$$x^2 - 4x - 12 \ge 0$$

$$(x - 6)(x + 2) \ge 0$$

$y \ge 7$ when $x \le -2, x \ge 6$.

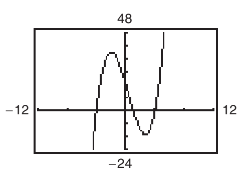
35. $y = \frac{1}{8}x^3 - \frac{1}{2}x$



(a) $y \ge 0$ when $-2 \le x \le 0, 2 \le x < \infty$.

(b) $y \le 6$ when $x \le 4$.

36. $y = x^3 - x^2 - 16x + 16$



(a) $y \le 0$

$$x^3 - x^2 - 16x + 16 \le 0$$

$$x^2(x - 1) - 16(x - 1) \le 0$$

$$(x - 1)(x^2 - 16) \le 0$$

$y \le 0$ when $-\infty < x \le -4, 1 \le x \le 4$.

(b) $y \ge 36$

$$x^3 - x^2 - 16x + 16 \ge 36$$

$$x^3 - x^2 - 16x - 20 \ge 0$$

$$(x + 2)(x - 5)(x + 2) \ge 0$$

$y \ge 36$ when $x = -2, 5 \le x < \infty$.

37. $\frac{1}{x} - x > 0$

$$\frac{1 - x^2}{x} > 0$$

Critical numbers: $x = 0, x = \pm 1$

Test intervals: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$

Test: Is $\frac{1 - x^2}{x} > 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(-\infty, -1) \cup (0, 1)$

38. $\frac{1}{x} - 4 < 0$

$$\frac{1 - 4x}{x} < 0$$

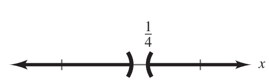
Critical numbers: $x = 0, x = \frac{1}{4}$

Test intervals: $(-\infty, 0) \Rightarrow \frac{1 - 4x}{x} < 0$

$$\left(0, \frac{1}{4}\right) \Rightarrow \frac{1 - 4x}{x} > 0$$

$$\left(\frac{1}{4}, \infty\right) \Rightarrow \frac{1 - 4x}{x} < 0$$

Solution interval: $(-\infty, 0) \cup \left(\frac{1}{4}, \infty\right)$



39. $$\frac{x+6}{x+1} - 2 < 0$$

$$\frac{x+6-2(x+1)}{x+1} < 0$$

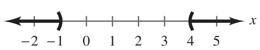
$$\frac{4-x}{x+1} < 0$$

Critical numbers: $x = -1, x = 4$

Test intervals: $(-\infty, -1), (-1, 4), (4, \infty)$

Test: Is $\frac{4-x}{x+1} < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(-\infty, -1) \cup (4, \infty)$



40. $$\frac{x+12}{x+2} - 3 \ge 0$$

$$\frac{x+12-3(x+2)}{x+2} \ge 0$$

$$\frac{6-2x}{x+2} \ge 0$$

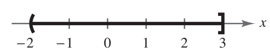
Critical numbers: $x = -2, x = 3$

Test intervals: $(-\infty, -2) \Rightarrow \frac{6-2x}{x+2} < 0$

$(-2, 3) \Rightarrow \frac{6-2x}{x+2} > 0$

$(3, \infty) \Rightarrow \frac{6-2x}{x+2} < 0$

Solution interval: $(-2, 3]$



41. $$\frac{3x-5}{x-5} > 4$$

$$\frac{3x-5}{x-5} - 4 > 0$$

$$\frac{3x-5-4(x-5)}{x-5} > 0$$

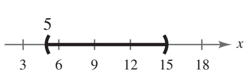
$$\frac{15-x}{x-5} > 0$$

Critical numbers: $x = 5, x = 15$

Test intervals: $(-\infty, 5), (5, 15), (15, \infty)$

Test: Is $\frac{15-x}{x-5} > 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(5, 15)$



42. $$\frac{5+7x}{1+2x} < 4$$

$$\frac{5+7x-4(1+2x)}{1+2x} < 0$$

$$\frac{1-x}{1+2x} < 0$$

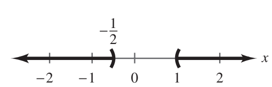
Critical numbers: $x = -\frac{1}{2}, x = 1$

Test intervals: $\left(-\infty, -\frac{1}{2}\right) \Rightarrow \frac{1-x}{1+2x} < 0$

$\left(-\frac{1}{2}, 1\right) \Rightarrow \frac{1-x}{1+2x} > 0$

$(1, \infty) \Rightarrow \frac{1-x}{1+2x} < 0$

Solution intervals: $\left(-\infty, -\frac{1}{2}\right) \cup (1, \infty)$



43. $$\frac{4}{x+5} > \frac{1}{2x+3}$$

$$\frac{4}{x+5} - \frac{1}{2x+3} > 0$$

$$\frac{4(2x+3)-(x+5)}{(x+5)(2x+3)} > 0$$

$$\frac{7x+7}{(x+5)(2x+3)} > 0$$

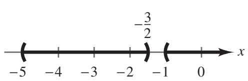
Critical numbers: $x = -1, x = -5, x = -\frac{3}{2}$

Test intervals: $(-\infty, -5), \left(-5, -\frac{3}{2}\right),$

$\left(-\frac{3}{2}, -1\right), (-1, \infty)$

Test: Is $\frac{7(x+1)}{(x+5)(2x+3)} > 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $\left(-5, -\frac{3}{2}\right) \cup (-1, \infty)$



44. $$\frac{5}{x-6} > \frac{3}{x+2}$$

$$\frac{5(x+2)-3(x-6)}{(x-6)(x+2)} > 0$$

$$\frac{2x+28}{(x-6)(x+2)} > 0$$

Critical numbers: $x = -14, x = -2, x = 6$

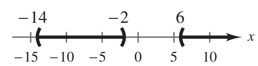
Test intervals: $(-\infty, -14) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(-14, -2) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

$(-2, 6) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(6, \infty) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

Solution intervals: $(-14, -2) \cup (6, \infty)$



45. $$\frac{1}{x-3} \le \frac{9}{4x+3}$$

$$\frac{1}{x-3} - \frac{9}{4x+3} \le 0$$

$$\frac{4x+3-9(x-3)}{(x-3)(4x+3)} \le 0$$

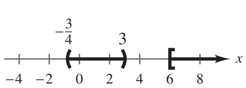
$$\frac{30-5x}{(x-3)(4x+3)} \le 0$$

Critical numbers: $x = 3, x = -\frac{3}{4}, x = 6$

Test intervals: $\left(-\infty, -\frac{3}{4}\right), \left(-\frac{3}{4}, 3\right), (3, 6), (6, \infty)$

Test: Is $\frac{30-5x}{(x-3)(4x+3)} \le 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $\left(-\frac{3}{4}, 3\right) \cup [6, \infty)$



46. $$\frac{1}{x} \ge \frac{1}{x+3}$$

$$\frac{1(x+3)-1(x)}{x(x+3)} \ge 0$$

$$\frac{3}{x(x+3)} \ge 0$$

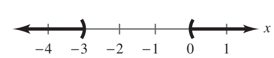
Critical numbers: $x = -3, x = 0$

Test intervals: $(-\infty, -3) \Rightarrow \frac{3}{x(x+3)} > 0$

$(-3, 0) \Rightarrow \frac{3}{x(x+3)} < 0$

$(0, \infty) \Rightarrow \frac{3}{x(x+3)} > 0$

Solution intervals: $(-\infty, -3) \cup (0, \infty)$



47. $$\frac{x^2 + 2x}{x^2 - 9} \le 0$$

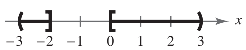
$$\frac{x(x + 2)}{(x + 3)(x - 3)} \le 0$$

Critical numbers: $x = 0, x = -2, x = \pm 3$

Test intervals:
$(-\infty, -3), (-3, -2), (-2, 0), (0, 3), (3, \infty)$

Test: Is $\dfrac{x(x + 2)}{(x + 3)(x - 3)} \le 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(-3, -2] \cup [0, 3)$



48. $$\frac{x^2 + x - 6}{x} \ge 0$$

$$\frac{(x + 3)(x - 2)}{x} \ge 0$$

Critical numbers: $x = -3, x = 0, x = 2$

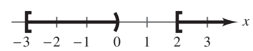
Test intervals: $(-\infty, -3) \Rightarrow \dfrac{(x + 3)(x - 2)}{x} < 0$

$(-3, 0) \Rightarrow \dfrac{(x + 3)(x - 2)}{x} > 0$

$(0, 2) \Rightarrow \dfrac{(x + 3)(x - 2)}{x} < 0$

$(2, \infty) \Rightarrow \dfrac{(x + 3)(x - 2)}{x} > 0$

Solution intervals: $[-3, 0) \cup [2, \infty)$



49. $$\frac{5}{x - 1} - \frac{2x}{x + 1} < 1$$

$$\frac{5}{x - 1} - \frac{2x}{x + 1} - 1 < 0$$

$$\frac{5(x + 1) - 2x(x - 1) - (x - 1)(x + 1)}{(x - 1)(x + 1)} < 0$$

$$\frac{5x + 5 - 2x^2 + 2x - x^2 + 1}{(x - 1)(x + 1)} < 0$$

$$\frac{-3x^2 + 7x + 6}{(x - 1)(x + 1)} < 0$$

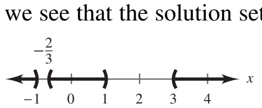
$$\frac{-(3x + 2)(x - 3)}{(x - 1)(x + 1)} < 0$$

Critical numbers: $x = -\dfrac{2}{3}, x = 3, x = \pm 1$

Test intervals: $(-\infty, -1), \left(-1, -\dfrac{2}{3}\right), \left(-\dfrac{2}{3}, 1\right), (1, 3), (3, \infty)$

Test: Is $\dfrac{-(3x + 2)(x - 3)}{(x - 1)(x + 1)} < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(-\infty, -1) \cup \left(-\frac{2}{3}, 1\right) \cup (3, \infty)$



50. $$\frac{3x}{x - 1} \le \frac{x}{x + 4} + 3$$

$$\frac{3x(x + 4) - x(x - 1) - 3(x + 4)(x - 1)}{(x - 1)(x + 4)} \le 0$$

$$\frac{-x^2 + 4x + 12}{(x - 1)(x + 4)} \le 0$$

$$\frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} \le 0$$

Critical numbers: $x = -4, x = -2, x = 1, x = 6$

Test intervals: $(-\infty, -4) \Rightarrow \dfrac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0$

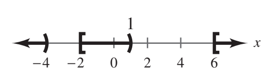
$(-4, -2) \Rightarrow \dfrac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} > 0$

$(-2, 1) \Rightarrow \dfrac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0$

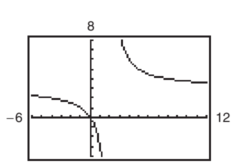
$(1, 6) \Rightarrow \dfrac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} > 0$

$(6, \infty) \Rightarrow \dfrac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0$

Solution intervals: $(-\infty, -4), [-2, 1), [6, \infty)$



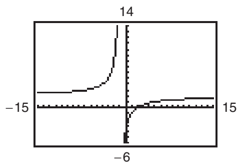
51. $y = \dfrac{3x}{x - 2}$



(a) $y \le 0$ when $0 \le x < 2$.

(b) $y \ge 6$ when $2 < x \le 4$.

52. $y = \dfrac{2(x-2)}{x+1}$



(a) $y \le 0$

$$\frac{2(x-2)}{x+1} \le 0$$

$y \le 0$ when $-1 < x \le 2$.

(b) $y \ge 8$

$$\frac{2(x-2)}{x+1} \ge 8$$

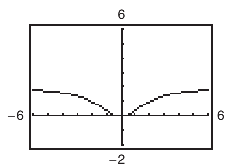
$$\frac{2(x-2) - 8(x+1)}{x+1} \ge 0$$

$$\frac{-6x-12}{x+1} \ge 0$$

$$\frac{-6(x+2)}{x+1} \ge 0$$

$y \ge 8$ when $-2 \le x < -1$.

53. $y = \dfrac{2x^2}{x^2+4}$



(a) $y \ge 1$ when $x \le -2$ or $x \ge 2$.

This can also be expressed as $|x| \ge 2$.

(b) $y \le 2$ for all real numbers x.

This can also be expressed as $-\infty < x < \infty$.

54. $y = \dfrac{5x}{x^2+4}$

(a) $y \ge 1$

$$\frac{5x}{x^2+4} \ge 1$$

$$\frac{5x-(x^2+4)}{x^2+4} \ge 0$$

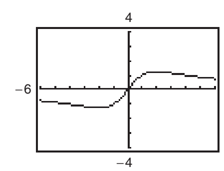
$$\frac{-(x-4)(x-1)}{x^2+4} \ge 0$$

$y \ge 1$ when $1 \le x \le 4$.

(b) $y \le 0$

$$\frac{5x}{x^2+4} \le 0$$

$y \le 0$ when $-\infty < x \le 0$.



55. $4 - x^2 \ge 0$

$(2+x)(2-x) \ge 0$

Critical numbers: $x = \pm 2$

Test intervals: $(-\infty, -2), (-2, 2), (2, \infty)$

Test: Is $4 - x^2 \ge 0$?

By testing an x-value in each test interval in the inequality, we see that the domain set is: $[-2, 2]$

56. $x^2 - 4 \ge 0$

$(x+2)(x-2) \ge 0$

Critical numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \implies (x+2)(x-2) > 0$

$(-2, 2) \implies (x+2)(x-2) < 0$

$(2, \infty) \implies (x+2)(x-2) > 0$

Domain: $(-\infty, -2] \cup [2, \infty)$

57. $x^2 - 7x + 12 \ge 0$

$(x-3)(x-4) \ge 0$

Critical numbers: $x = 3, x = 4$

Test intervals: $(-\infty, 3), (3, 4), (4, \infty)$

Test: Is $(x-3)(x-4) \ge 0$?

By testing an x-value in each test interval in the inequality, we see that the domain set is: $(-\infty, 3] \cup [4, \infty)$

58. $144 - 9x^2 \ge 0$

$9(4-x)(4+x) \ge 0$

Critical numbers: $x = -4, x = 4$

Test intervals: $(-\infty, -4) \implies 9(4-x)(4+x) < 0$

$(-4, 4) \implies 9(4-x)(4+x) > 0$

$(4, \infty) \implies 9(4-x)(4+x) < 0$

Domain: $[-4, 4]$

59. $\dfrac{x}{x^2 - 2x - 35} \geq 0$

$\dfrac{x}{(x + 5)(x - 7)} \geq 0$

Critical numbers: $x = 0, x = -5, x = 7$

Test intervals: $(-\infty, -5), (-5, 0), (0, 7), (7, \infty)$

Test: Is $\dfrac{x}{(x + 5)(x - 7)} \geq 0$?

By testing an x-value in each test interval in the inequality, we see that the domain set is: $(-5, 0] \cup (7, \infty)$

60. $\dfrac{x}{x^2 - 9} \geq 0$

$\dfrac{x}{(x + 3)(x - 3)} \geq 0$

Critical numbers: $x = -3, x = 0, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(-3, 0) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

$(0, 3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

Domain: $(-3, 0] \cup (3, \infty)$

61. $0.4x^2 + 5.26 < 10.2$

$0.4x^2 - 4.94 < 0$

$0.4(x^2 - 12.35) < 0$

Critical numbers: $x \approx \pm 3.51$

Test intervals: $(-\infty, -3.51), (-3.51, 3.51), (3.51, \infty)$

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(-3.51, 3.51)$

62. $-1.3x^2 + 3.78 > 2.12$

$-1.3x^2 + 1.66 > 0$

Critical numbers: ± 1.13

Test intervals: $(-\infty, -1.13), (-1.13, 1.13), (1.13, \infty)$

Solution set: $(-1.13, 1.13)$

63. $-0.5x^2 + 12.5x + 1.6 > 0$

The zeros are $x = \dfrac{-12.5 \pm \sqrt{(12.5)^2 - 4(-0.5)(1.6)}}{2(-0.5)}$.

Critical numbers: $x \approx -0.13, x \approx 25.13$

Test intervals: $(-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)$

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(-0.13, 25.13)$

64. $1.2x^2 + 4.8x + 3.1 < 5.3$

$1.2x^2 + 4.8x - 2.2 < 0$

Critical numbers: $-4.42, 0.42$

Test intervals: $(-\infty, -4.42), (-4.42, 0.42), (0.42, \infty)$

Solution set: $(-4.42, 0.42)$

65. $\dfrac{1}{2.3x - 5.2} > 3.4$

$\dfrac{1}{2.3x - 5.2} - 3.4 > 0$

$\dfrac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0$

$\dfrac{-7.82x + 18.68}{2.3x - 5.2} > 0$

Critical numbers: $x \approx 2.39, x \approx 2.26$

Test intervals: $(-\infty, 2.26), (2.26, 2.39), (2.39, \infty)$

By testing an x-value in each test interval in the inequality, we see that the solution set is: $(2.26, 2.39)$

66. $\dfrac{2}{3.1x - 3.7} > 5.8$

$\dfrac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0$

$\dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

Critical numbers: $x \approx 1.19, x \approx 1.30$

Test intervals: $(-\infty, 1.19) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

$(1.19, 1.30) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

$(1.30, \infty) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

Solution interval: $(1.19, 1.30)$

67. $s = -16t^2 + v_0t + s_0 = -16t^2 + 160t$

(a) $-16t^2 + 160t = 0$

$-16t(t - 10) = 0$

$t = 0, t = 10$

It will be back on the ground in 10 seconds.

(b) $-16t^2 + 160t > 384$

$-16t^2 + 160t - 384 > 0$

$-16(t^2 - 10t + 24) > 0$

$t^2 - 10t + 24 < 0$

$(t - 4)(t - 6) < 0$

$4 < t < 6$ seconds

68. $s = -16t^2 + v_0t + s_0 = -16t^2 + 128t$

(a) $-16t^2 + 128t = 0$

$-16t(t - 8) = 0$

$-16t = 0 \implies t = 0$

$t - 8 = 0 \implies t = 8$

It will be back on the ground in 8 seconds.

(b) $-16t^2 + 128t < 128$

$-16t^2 + 128t - 128 < 0$

Critical numbers: $4 - 2\sqrt{2}, 4 + 2\sqrt{2}$

Test intervals:
$\left(-\infty, 4 - 2\sqrt{2}\right), \left(4 - 2\sqrt{2}, 4 + 2\sqrt{2}\right),$
$\left(4 + 2\sqrt{2}, \infty\right)$

Solution set: 0 seconds $\le t < 4 - 2\sqrt{2}$ seconds
and $4 - 2\sqrt{2}$ seconds $< t \le 8$ seconds

69. $2L + 2W = 100 \implies W = 50 - L$

$LW \ge 500$

$L(50 - L) \ge 500$

$-L^2 + 50L - 500 \ge 0$

By the Quadratic Formula we have:

Critical numbers: $L = 25 \pm 5\sqrt{5}$

Test: Is $-L^2 + 50L - 500 \ge 0$?

Solution set: $25 - 5\sqrt{5} \le L \le 25 + 5\sqrt{5}$

13.8 meters $\le L \le$ 36.2 meters

70. $2L + 2W = 440 \implies W = 220 - L$

$LW \ge 8000$

$L(220 - L) \ge 8000$

$-L^2 + 220L - 8000 \ge 0$

By the Quadratic Formula we have:

Critical numbers: $L = 110 \pm 10\sqrt{41}$

Test: Is $-L^2 + 220L - 8000 \ge 0$?

Solution set: $110 - 10\sqrt{41} \le L \le 110 + 10\sqrt{41}$

45.97 feet $\le L \le$ 174.03 feet

71. $R = x(75 - 0.0005x)$ and $C = 30x + 250{,}000$

$P = R - C$

$= (75x - 0.0005x^2) - (30x + 250{,}000)$

$= -0.0005x^2 + 45x - 250{,}000$

$P \ge 750{,}000$

$-0.0005x^2 + 45x - 250{,}000 \ge 750{,}000$

$-0.0005x^2 + 45x - 1{,}000{,}000 \ge 0$

Critical numbers: $x = 40{,}000, x = 50{,}000$ (These were obtained by using the Quadratic Formula.)

Test intervals: $(0, 40{,}000), (40{,}000, 50{,}000), (50{,}000, \infty)$

By testing x-values in each test interval in the inequality, we see that the solution set is $[40{,}000, 50{,}000]$ or $40{,}000 \le x \le 50{,}000$. The price per unit is

$$p = \frac{R}{x} = 75 - 0.0005x.$$

For $x = 40{,}000$, $p = \$55$. For $x = 50{,}000$, $p = \$50$. Therefore, for $40{,}000 \le x \le 50{,}000$, $\$50.00 \le p \le \$55.00$.

72. What is the price per unit?

When $x = 90{,}000$:

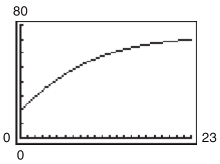
$R = \$2{,}880{,}000 \implies \dfrac{2{,}880{,}000}{90{,}000} = \$32$ per unit

When $x = 100{,}000$:

$R = \$3{,}000{,}000 \implies \dfrac{3{,}000{,}000}{100{,}000} = \$30$ per unit

Solution interval: $\$30.00 \le p \le \$32.00$

73. $C = 0.0031t^3 - 0.216t^2 + 5.54t + 19.1,\ 0 \le t \le 23$

(a) 

(b)

t	C
24	70.5
26	71.6
28	72.9
30	74.6
32	76.8
34	79.6

C will be greater than 75% when $t \approx 31$, which corresponds to 2011.

(c) $C = 75$ when $t \approx 30.41$.

(d)

t	C
36	83.2
37	85.4
38	87.8
39	90.5
40	93.5
41	96.8
42	100.4
43	104.4

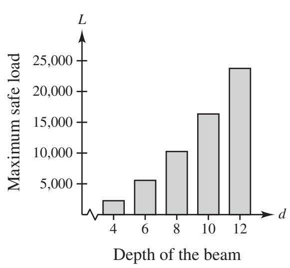
C will be between 85% and 100% when t is between 37 and 42. These values correspond to the years 2017 to 2022.

(e) $85 \le C \le 100$ when $36.82 \le t \le 41.89$ or $37 \le t \le 42$.

(f) The model is a third-degree polynomial and as $t \to \infty$, $C \to \infty$.

74. (a)

d	4	6	8	10	12
Load	2223.9	5593.9	10,312	16,378	23,792



(b) $2000 \le 168.5d^2 - 472.1$

$2472.1 \le 168.5d^2$

$14.67 \le d^2$

$3.83 \le d$

The minimum depth is 3.83 inches.

75. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{2}$

$2R_1 = 2R + RR_1$

$2R_1 = R(2 + R_1)$

$\dfrac{2R_1}{2 + R_1} = R$

Since $R \ge 1$, we have

$$\frac{2R_1}{2 + R_1} \ge 1$$

$$\frac{2R_1}{2 + R_1} - 1 \ge 0$$

$$\frac{R_1 - 2}{2 + R_1} \ge 0.$$

Since $R_1 > 0$, the only critical number is $R_1 = 2$. The inequality is satisfied when $R_1 \ge 2$ ohms.

76. (a) $N = -0.03t^2 + 9.6t + 172$

$= 220 \implies t = 5$

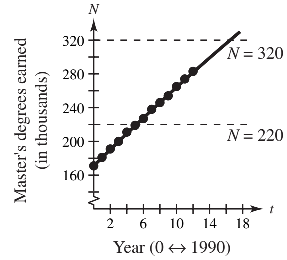
So the number of master's degrees earned by women exceeded 220,000 in 1995.

(c) $N = -0.03t^2 + 9.6t + 172$

$= 320 \implies t = 16.2$

So the number of master's degrees earned by women will exceed 320,000 in 2006.

(b) and (d)



77. True

$x^3 - 2x^2 - 11x + 12 = (x + 3)(x - 1)(x - 4)$

The test intervals are $(-\infty, -3)$, $(-3, 1)$, $(1, 4)$, and $(4, \infty)$.

78. True

The y-values are greater than zero for all values of x.

79. $x^2 + bx + 4 = 0$

To have at least one real solution, $b^2 - 16 \geq 0$. This occurs when $b \leq -4$ or $b \geq 4$. This can be written as $(-\infty, -4] \cup [4, \infty)$.

80. $x^2 + bx - 4 = 0$

To have at least one real solution,

$$b^2 - 4(1)(-4) \geq 0$$

$$b^2 + 16 \geq 0.$$

This inequality is true for all real values of b. Thus, the interval for b such that the equation has at least one real solution is $(-\infty, \infty)$.

81. $3x^2 + bx + 10 = 0$

To have at least one real solution, $b^2 - 4(3)(10) \geq 0$.

$$b^2 - 120 \geq 0$$

$$\left(b + \sqrt{120}\right)\left(b - \sqrt{120}\right) \geq 0$$

Critical numbers: $b = \pm\sqrt{120} = \pm 2\sqrt{30}$

Test intervals:
$\left(-\infty, -2\sqrt{30}\right)$, $\left(-2\sqrt{30}, 2\sqrt{30}\right)$, $\left(2\sqrt{30}, \infty\right)$

Test: Is $b^2 - 120 \geq 0$?

Solution set: $\left(-\infty, -2\sqrt{30}\right] \cup \left[2\sqrt{30}, \infty\right)$

82. $2x^2 + bx + 5 = 0$

To have at least one real solution,

$$b^2 - 4(2)(5) \geq 0$$

$$b^2 - 40 \geq 0.$$

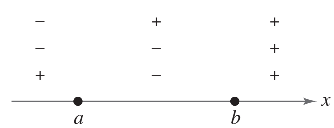
This occurs when $b \leq -2\sqrt{10}$ or $b \geq 2\sqrt{10}$. Thus, the interval for b such that the equation has at least one real solution is $\left(-\infty, -2\sqrt{10}\right] \cup \left[2\sqrt{10}, \infty\right)$.

83. (a) If $a > 0$ and $c \leq 0$, then b can be any real number. If $a > 0$ and $c > 0$, then for $b^2 - 4ac$ to be greater than or equal to zero, b is restricted to $b < -2\sqrt{ac}$ or $b > 2\sqrt{ac}$.

(b) The center of the interval for b in Exercises 79–82 is 0.

84. (a) $x = a, x = b$

(b)



(c) The real zeros of the polynomial

85. $4x^2 + 20x + 25 = (2x + 5)^2$

86. $(x + 3)^2 - 16 = [(x + 3) + 4][(x + 3) - 4]$
$= (x + 7)(x - 1)$

87. $x^2(x + 3) - 4(x + 3) = (x^2 - 4)(x + 3)$
$= (x + 2)(x - 2)(x + 3)$

88. $2x^4 - 54x = 2x(x^3 - 27)$
$= 2x(x - 3)(x^2 + 3x + 9)$

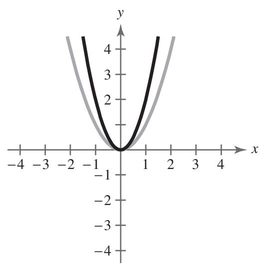
89. Area $=$ (length)(width)
$= (2x + 1)(x)$
$= 2x^2 + x$

90. Area $= \frac{1}{2}$(base)(height)
$= \frac{1}{2}(b)(3b + 2)$
$= \frac{3}{2}b^2 + b$

Review Exercises for Chapter 2

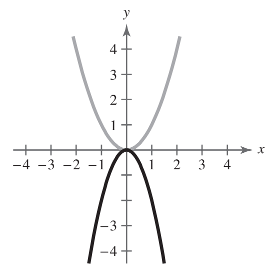
1. (a) $y = 2x^2$

Vertical stretch



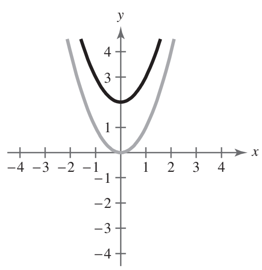
(b) $y = -2x^2$

Vertical stretch and a reflection in the x-axis



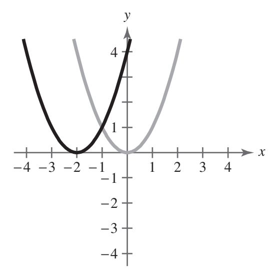
(c) $y = x^2 + 2$

Vertical shift two units upward



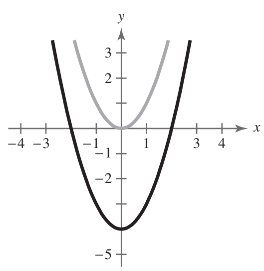
(d) $y = (x + 2)^2$

Horizontal shift two units to the left



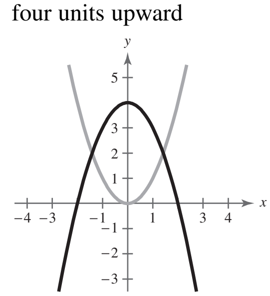
2. (a) $y = x^2 - 4$

Vertical shift four units downward



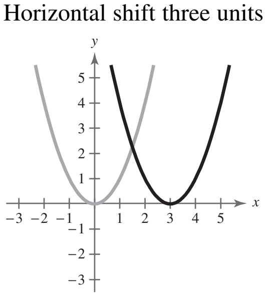
(b) $y = 4 - x^2$

Reflection in the x-axis and a vertical shift four units upward



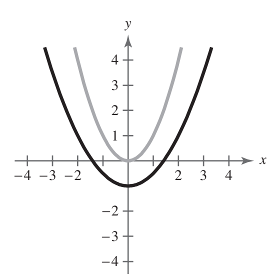
(c) $y = (x - 3)^2$

Horizontal shift three units to the right



(d) $y = \frac{1}{2}x^2 - 1$

Vertical shrink (each y-value is multiplied by $\frac{1}{2}$), and a vertical shift one unit downward



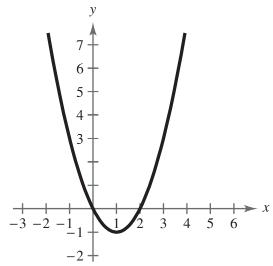
3. $$\begin{aligned} g(x) &= x^2 - 2x \\ &= x^2 - 2x + 1 - 1 \\ &= (x - 1)^2 - 1 \end{aligned}$$

Vertex: $(1, -1)$

Axis of symmetry: $x = 1$

$0 = x^2 - 2x = x(x - 2)$

x-intercepts: $(0, 0)$, $(2, 0)$



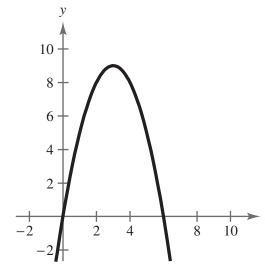
4. $$\begin{aligned} f(x) &= 6x - x^2 \\ &= -(x^2 - 6x + 9 - 9) \\ &= -(x - 3)^2 + 9 \end{aligned}$$

Vertex: $(3, 9)$

Axis of symmetry: $x = 3$

$0 = 6x - x^2 = x(6 - x)$

x-intercepts: $(0, 0)$, $(6, 0)$



5. $$\begin{aligned} f(x) &= x^2 + 8x + 10 \\ &= x^2 + 8x + 16 - 16 + 10 \\ &= (x + 4)^2 - 6 \end{aligned}$$

Vertex: $(-4, -6)$

Axis of symmetry: $x = -4$

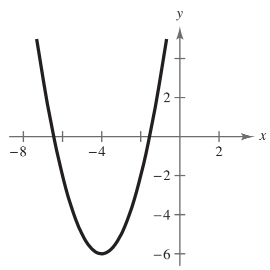
$$0 = (x + 4)^2 - 6$$

$(x + 4)^2 = 6$

$$x + 4 = \pm\sqrt{6}$$

$$x = -4 \pm \sqrt{6}$$

x-intercepts: $\left(-4 \pm \sqrt{6}, 0\right)$



6. $$\begin{aligned} h(x) &= 3 + 4x - x^2 \\ &= -(x^2 - 4x - 3) \\ &= -(x^2 - 4x + 4 - 4 - 3) \\ &= -[(x - 2)^2 - 7] \\ &= -(x - 2)^2 + 7 \end{aligned}$$

Vertex: $(2, 7)$

Axis of symmetry: $x = 2$

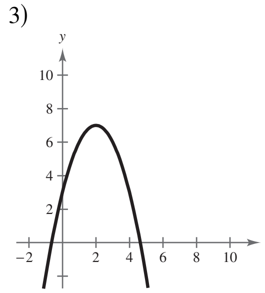
$0 = 3 + 4x - x^2$

$0 = x^2 - 4x - 3$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{28}}{2} = 2 \pm \sqrt{7}$$

x-intercepts: $\left(2 \pm \sqrt{7}, 0\right)$



7. $$\begin{aligned} f(t) &= -2t^2 + 4t + 1 \\ &= -2(t^2 - 2t + 1 - 1) + 1 \\ &= -2[(t - 1)^2 - 1] + 1 \\ &= -2(t - 1)^2 + 3 \end{aligned}$$

Vertex: $(1, 3)$

Axis of symmetry: $t = 1$

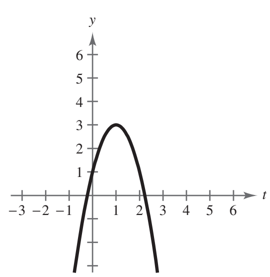
$$0 = -2(t - 1)^2 + 3$$

$2(t - 1)^2 = 3$

$$t - 1 = \pm\sqrt{\frac{3}{2}}$$

$$t = 1 \pm \frac{\sqrt{6}}{2}$$

t-intercepts: $\left(1 \pm \dfrac{\sqrt{6}}{2}, 0\right)$



8. $$\begin{aligned} f(x) &= x^2 - 8x + 12 \\ &= x^2 - 8x + 16 - 16 + 12 \\ &= (x - 4)^2 - 4 \end{aligned}$$

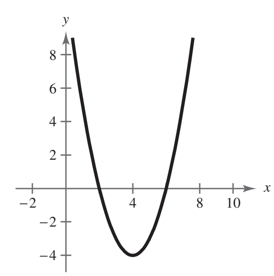
Vertex: $(4, -4)$

Axis of symmetry: $x = 4$

$0 = x^2 - 8x + 12$

$0 = (x - 2)(x - 6)$

x-intercepts: $(2, 0)$, $(6, 0)$



9. $h(x) = 4x^2 + 4x + 13$

$= 4(x^2 + x) + 13$

$= 4\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + 13$

$= 4\left(x^2 + x + \frac{1}{4}\right) - 1 + 13$

$= 4\left(x + \frac{1}{2}\right)^2 + 12$

Vertex: $\left(-\frac{1}{2}, 12\right)$

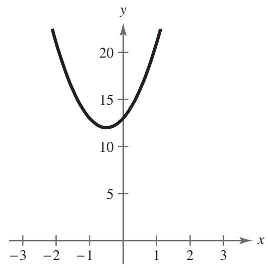
Axis of symmetry: $x = -\frac{1}{2}$

$0 = 4\left(x + \frac{1}{2}\right)^2 + 12$

$\left(x + \frac{1}{2}\right)^2 = -3$

No real zeros

x-intercepts: none



10. $f(x) = x^2 - 6x + 1$

$= x^2 - 6x + 9 - 9 + 1$

$= (x - 3)^2 - 8$

Vertex: $(3, -8)$

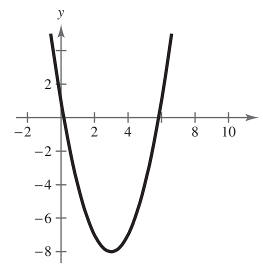
Axis of symmetry: $x = 3$

$0 = x^2 - 6x + 1$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)}$

$= \frac{6 \pm \sqrt{32}}{2} = 3 \pm 2\sqrt{2}$

x-intercepts: $\left(3 \pm 2\sqrt{2}, 0\right)$



11. $h(x) = x^2 + 5x - 4$

$= x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 4$

$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{16}{4}$

$= \left(x + \frac{5}{2}\right)^2 - \frac{41}{4}$

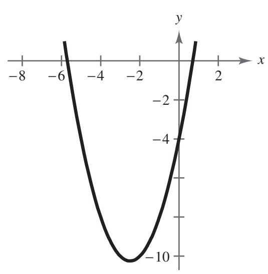
Vertex: $\left(-\frac{5}{2}, -\frac{41}{4}\right)$

Axis of symmetry: $x = -\frac{5}{2}$

$0 = x^2 + 5x - 4$

By the Quadratic Formula, $x = \frac{-5 \pm \sqrt{41}}{2}$.

x-intercepts: $\left(\frac{-5 \pm \sqrt{41}}{2}, 0\right)$



12. $f(x) = 4x^2 + 4x + 5$

$= 4\left(x^2 + x + \frac{1}{4} - \frac{1}{4} + \frac{5}{4}\right)$

$= 4\left[\left(x + \frac{1}{2}\right)^2 + 1\right]$

$= 4\left(x + \frac{1}{2}\right)^2 + 4$

Vertex: $\left(-\frac{1}{2}, 4\right)$

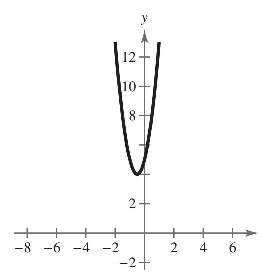
Axis of symmetry: $x = -\frac{1}{2}$

$0 = 4x^2 + 4x + 5$

By the Quadratic Formula, $x = \frac{-4 \pm 8i}{8} = -\frac{1}{2} \pm i$.

The equation has no real zeros.

x-intercepts: None



13. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$

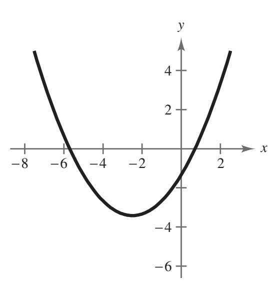
$= \frac{1}{3}\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 4\right)$

$= \frac{1}{3}\left[\left(x + \frac{5}{2}\right)^2 - \frac{41}{4}\right]$

$= \frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12}$

Vertex: $\left(-\frac{5}{2}, -\frac{41}{12}\right)$

Axis of symmetry: $x = -\frac{5}{2}$



$0 = x^2 + 5x - 4$

By the Quadratic Formula, $x = \frac{-5 \pm \sqrt{41}}{2}$.

x-intercepts: $\left(\frac{-5 \pm \sqrt{41}}{2}, 0\right)$

14. $f(x) = \frac{1}{2}(6x^2 - 24x + 22)$

$= 3x^2 - 12x + 11$

$= 3(x^2 - 4x + 4 - 4) + 11$

$= 3(x - 2)^2 + 3(-4) + 11$

$= 3(x - 2)^2 - 1$

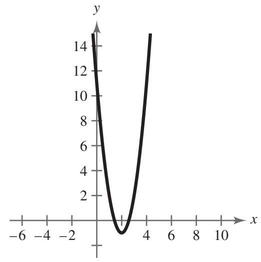
Vertex: $(2, -1)$

Axis of symmetry: $x = 2$

$0 = 3x^2 - 12x + 11$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(11)}}{2(3)} = \frac{12 \pm \sqrt{12}}{6} = 2 \pm \frac{\sqrt{3}}{3}$$

x-intercepts: $\left(2 \pm \frac{\sqrt{3}}{3}, 0\right)$



15. Vertex: $(4, 1) \implies f(x) = a(x - 4)^2 + 1$

Point: $(2, -1) \implies -1 = a(2 - 4)^2 + 1$

$-2 = 4a$

$-\frac{1}{2} = a$

Thus, $f(x) = -\frac{1}{2}(x - 4)^2 + 1$.

16. Vertex: $(2, 2) \implies f(x) = a(x - 2)^2 + 2$

Point: $(0, 3) \implies 3 = a(0 - 2)^2 + 2$

$3 = 4a + 2$

$1 = 4a$

$\frac{1}{4} = a$

$f(x) = \frac{1}{4}(x - 2)^2 + 2$

17. Vertex: $(1, -4) \implies f(x) = a(x - 1)^2 - 4$

Point: $(2, -3) \implies -3 = a(2 - 1)^2 - 4$

$1 = a$

Thus, $f(x) = (x - 1)^2 - 4$.

18. Vertex: $(2, 3) \implies f(x) = a(x - 2)^2 + 3$

Point: $(-1, 6) \implies 6 = a(-1 - 2)^2 + 3$

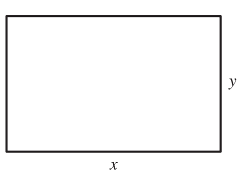
$6 = 9a + 3$

$3 = 9a$

$\frac{1}{3} = a$

$f(x) = \frac{1}{3}(x - 2)^2 + 3$

19. (a)



(b) $2x + 2y = 200$

$x + y = 100$

$y = 100 - x$

Area $= xy$

$= x(100 - x)$

$= 100x - x^2$

(c) Area $= 100x - x^2$

$= -(x^2 - 100x + 2500 - 2500)$

$= -[(x - 50)^2 - 2500]$

$= -(x - 50)^2 + 2500$

The maximum area occurs at the vertex when $x = 50$ and $y = 100 - 50 = 50$. The dimensions with the maximum area are $x = 50$ meters and $y = 50$ meters.

20. $R = -10p^2 + 800p$

(a) $R(20) = \$12{,}000$

$R(25) = \$13{,}750$

$R(30) = \$15{,}000$

(b) The maximum revenue occurs at the vertex of the parabola.

$$-\frac{b}{2a} = \frac{-800}{2(-10)} = \$40$$

$R(40) = \$16{,}000$

The revenue is maximum when the price is \$40 per unit.
The maximum revenue is \$16,000.

21. $C = 70{,}000 - 120x + 0.055x^2$

The minimum cost occurs at the vertex of the parabola.

Vertex: $-\dfrac{b}{2a} = -\dfrac{-120}{2(0.055)} \approx 1091$ units

Approximately 1091 units should be produced each day to yield a minimum cost.

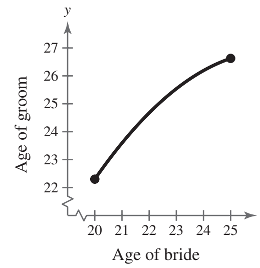
22. $26 = -0.107x^2 + 5.68x - 48.5$

$0 = -0.107x^2 + 5.68x - 74.5$

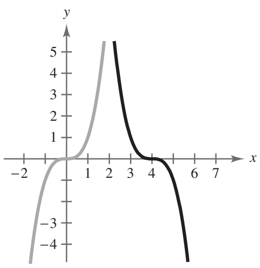
$$x = \frac{-5.68 \pm \sqrt{(5.68)^2 - 4(-0.107)(-74.5)}}{2(-0.107)}$$

$x \approx 23.7, 29.4$

The age of the bride is approximately 24 years when the age of the groom is 26 years.

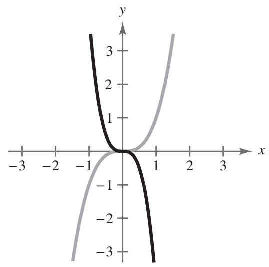


23. $y = x^3, f(x) = -(x - 4)^3$



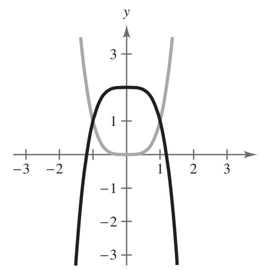
Transformation: Reflection in the x-axis and a horizontal shift four units to the right

24. $y = x^3, f(x) = -4x^3$



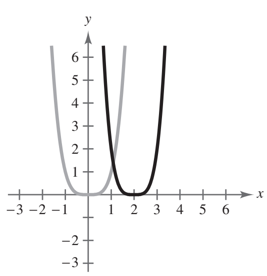
$f(x)$ is a reflection in the x-axis and a vertical stretch of the graph of $y = x^3$.

25. $y = x^4, f(x) = 2 - x^4$



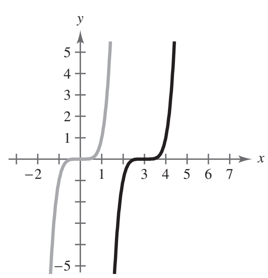
Transformation: Reflection in the x-axis and a vertical shift two units upward

26. $y = x^4, f(x) = 2(x - 2)^4$



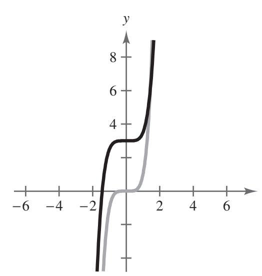
$f(x)$ is a shift to the right two units and a vertical stretch of the graph of $y = x^4$.

27. $y = x^5, f(x) = (x - 3)^5$



Transformation: Horizontal shift three units to the right

28. $y = x^5, f(x) = \frac{1}{2}x^5 + 3$



$f(x)$ is a vertical shrink and a vertical shift three units upward of the graph of $y = x^5$.

29. $f(x) = -x^2 + 6x + 9$

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

30. $f(x) = \frac{1}{2}x^3 + 2x$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

31. $g(x) = \frac{3}{4}(x^4 + 3x^2 + 2)$

The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

32. $h(x) = -x^5 - 7x^2 + 10x$

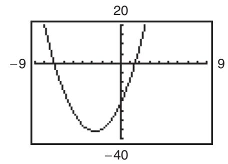
The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

33. $f(x) = 2x^2 + 11x - 21$

$$0 = 2x^2 + 11x - 21$$
$$= (2x - 3)(x + 7)$$

Zeros: $x = \frac{3}{2}, -7$, all of multiplicity 1 (odd multiplicity)

Turning points: 1



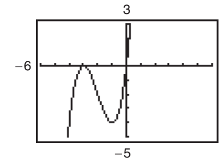
34. $f(x) = x(x + 3)^2$

$$0 = x(x + 3)^2$$

Zeros: $x = 0$ of multiplicity 1 (odd multiplicity)

$x = -3$ of multiplicity 2 (even multiplicity)

Turning points: 2

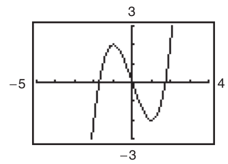


35. $f(t) = t^3 - 3t$

$$0 = t^3 - 3t$$
$$0 = t(t^2 - 3)$$

Zeros: $t = 0, \pm\sqrt{3}$ all of multiplicity 1 (odd multiplicity)

Turning points: 2



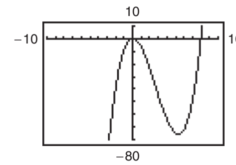
36. $f(x) = x^3 - 8x^2$

$$0 = x^3 - 8x^2$$
$$0 = x^2(x - 8)$$

Zeros: $x = 0$ of multiplicity 2 (even multiplicity)

$x = 8$ of multiplicity 1 (odd multiplicity)

Turning points: 2



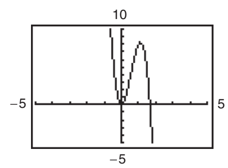
37. $f(x) = -12x^3 + 20x^2$

$$0 = -12x^3 + 20x^2$$
$$0 = -4x^2(3x - 5)$$

Zeros: $x = 0$ of multiplicity 2 (even multiplicity)

$x = \frac{5}{3}$ of multiplicity 1 (odd multiplicity)

Turning points: 2



38. $g(x) = x^4 - x^3 - 2x^2$

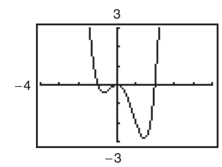
$$0 = x^4 - x^3 - 2x^2$$
$$0 = x^2(x^2 - x - 2)$$
$$= x^2(x + 1)(x - 2)$$

Zeros: $x = 0$ of multiplicity 2 (even multiplicity)

$x = -1$ of multiplicity 1 (odd multiplicity)

$x = 2$ of multiplicity 1 (odd multiplicity)

Turning points: 3



39. $f(x) = -x^3 + x^2 - 2$

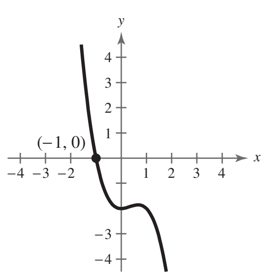
(a) The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

(b) Zero: $x = -1$

(c)

x	-3	-2	-1	0	1	2
$f(x)$	34	10	0	-2	-2	-6

(d)



40. $g(x) = 2x^3 + 4x^2$

(a) The degree is odd and the leading coefficient, 2, is positive. The graph rises to the right and falls to the left.

(b) $g(x) = 2x^3 + 4x^2$

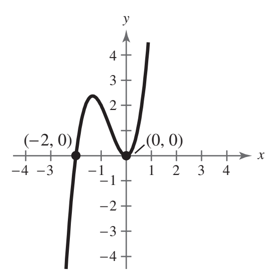
$$0 = 2x^3 + 4x^2$$
$$0 = 2x^2(x + 2)$$
$$0 = x^2(x + 2)$$

The zeros are 0 and -2.

(c)

x	-3	-2	-1	0	1
$g(x)$	-18	0	2	0	6

(d)



41. $f(x) = x(x^3 + x^2 - 5x + 3)$

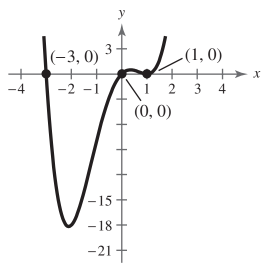
(a) The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

(b) Zeros: $x = 0, 1, -3$

(c)

x	-4	-3	-2	-1	0	1	2	3
$f(x)$	100	0	-18	-8	0	0	10	72

(d)



42. $h(x) = 3x^2 - x^4$

(a) The degree is even and the leading coefficient, -1, is negative. The graph falls to the left and falls to the right.

(b) $g(x) = 3x^2 - x^4$

$0 = 3x^2 - x^4$

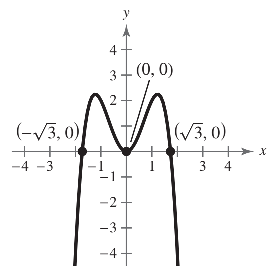
$0 = x^2(3 - x^2)$

The zeros are 0, $-\sqrt{3}$, and $\sqrt{3}$.

(c)

x	-2	-1	0	1	2
$h(x)$	-4	2	0	2	-4

(d)



43. (a) $f(x) = 3x^3 - x^2 + 3$

x	-3	-2	-1	0	1	2	3
$f(x)$	-87	-25	-1	3	5	23	75

(b) The zero is in the interval $[-1, 0]$.

Zero: $x \approx -0.900$

44. (a) $f(x) = 0.25x^3 - 3.65x + 6.12$

x	-6	-5	-4	-3	-2
$f(x)$	-25.98	-6.88	4.72	10.32	11.42

x	-1	0	1	2	3	4
$f(x)$	9.52	6.12	2.72	0.82	1.92	7.52

(b) The only zero is in the interval $(-5, -4)$. It is $x \approx -4.479$.

45. (a) $f(x) = x^4 - 5x - 1$

x	-3	-2	-1	0	1	2	3
$f(x)$	95	25	5	-1	-5	5	65

(b) There are two zeros, one in the interval $[-1, 0]$ and one in the interval $[1, 2]$

Zeros: $x \approx -0.200, x \approx 1.772$

46. (a) $f(x) = 7x^4 + 3x^3 - 8x^2 + 2$

x	-3	-2	-1	0	1	2
$f(x)$	416	58	-2	2	4	106

(b) There are zeros in the intervals $(-2, -1)$ and $(-1, 0)$. They are $x \approx -1.211$ and $x \approx -0.509$.

47.

$$\begin{array}{r@{}l} & \quad 8x + 5 \\ 3x - 2 \,\big) & \overline{24x^2 - x - 8} \\ & \underline{24x^2 - 16x} \\ & \qquad 15x - 8 \\ & \qquad \underline{15x - 10} \\ & \qquad\qquad 2 \end{array}$$

Thus, $\dfrac{24x^2 - x - 8}{3x - 2} = 8x + 5 + \dfrac{2}{3x - 2}$.

48.

$$\begin{array}{r@{}l} & \quad \frac{4}{3} \\ 3x - 2 \,\big) & \overline{4x + 7} \\ & \underline{4x - \frac{8}{3}} \\ & \quad \frac{29}{3} \end{array}$$

$$\frac{4x + 7}{3x - 2} = \frac{4}{3} + \frac{29}{3(3x - 2)}$$

49.

$$\begin{array}{r} 5x + 2 \\ x^2 - 3x + 1\,\overline{)\,5x^3 - 13x^2 - x + 2} \\ \underline{5x^3 - 15x^2 + 5x} \\ 2x^2 - 6x + 2 \\ \underline{2x^2 - 6x + 2} \\ 0 \end{array}$$

Thus, $\dfrac{5x^3 - 13x^2 - x + 2}{x^2 - 3x + 1} = 5x + 2.$

50.

$$\begin{array}{r} 3x^2 \qquad + 3 \\ x^2 - 1\,\overline{)\,3x^4 + 0x^3 + 0x^2 + 0x + 0} \\ \underline{3x^4 \qquad - 3x^2} \\ 3x^2 \qquad + 0 \\ \underline{3x^2 \qquad - 3} \\ 3 \end{array}$$

$\dfrac{3x^4}{x^2 - 1} = 3x^2 + 3 + \dfrac{3}{x^2 - 1}$

51.

$$\begin{array}{r} x^2 - 3x + 2 \\ x^2 + 0x + 2\,\overline{)\,x^4 - 3x^3 + 4x^2 - 6x + 3} \\ \underline{x^4 + 0x^3 + 2x^2} \\ -3x^3 + 2x^2 - 6x \\ \underline{-3x^3 + 0x^2 - 6x} \\ 2x^2 + 0x + 3 \\ \underline{2x^2 + 0x + 4} \\ -1 \end{array}$$

Thus, $\dfrac{x^4 - 3x^3 + 4x^2 - 6x + 3}{x^2 + 2} = x^2 - 3x + 2 - \dfrac{1}{x^2 + 2}.$

52.

$$\begin{array}{r} 3x^2 + 5x + 8 \\ 2x^2 + 0x - 1\,\overline{)\,6x^4 + 10x^3 + 13x^2 - 5x + 2} \\ \underline{6x^4 + 0x^3 - 3x^2} \\ 10x^3 + 16x^2 - 5x \\ \underline{10x^3 + 0x^2 - 5x} \\ 16x^2 - 0x + 2 \\ \underline{16x^2 + 0x - 8} \\ 10 \end{array}$$

$\dfrac{6x^4 + 10x^3 + 13x^2 - 5x + 2}{2x^2 - 1} = 3x^2 + 5x + 8 + \dfrac{10}{2x^2 - 1}$

53.

$$\begin{array}{r|rrrrr} 2 & 6 & -4 & -27 & 18 & 0 \\ & & 12 & 16 & -22 & -8 \\ \hline & 6 & 8 & -11 & -4 & -8 \end{array}$$

Thus,

$\dfrac{6x^4 - 4x^3 - 27x^2 + 18x}{x - 2} = 6x^3 + 8x^2 - 11x - 4 - \dfrac{8}{x - 2}.$

54.

$$\begin{array}{r|rrrr} 5 & 0.1 & 0.3 & 0 & -0.5 \\ & & 0.5 & 4 & 20 \\ \hline & 0.1 & 0.8 & 4 & 19.5 \end{array}$$

$\dfrac{0.1x^3 + 0.3x^2 - 0.5}{x - 5} = 0.1x^2 + 0.8x + 4 + \dfrac{19.5}{x - 5}$

55.

$$\begin{array}{r|rrrr} 4 & 2 & -19 & 38 & 24 \\ & & 8 & -44 & -24 \\ \hline & 2 & -11 & -6 & 0 \end{array}$$

Thus, $\dfrac{2x^3 - 19x^2 + 38x + 24}{x - 4} = 2x^2 - 11x - 6.$

56.

$$\begin{array}{r|rrrr} -3 & 3 & 20 & 29 & -12 \\ & & -9 & -33 & 12 \\ \hline & 3 & 11 & -4 & 0 \end{array}$$

$\dfrac{3x^3 + 20x^2 + 29x - 12}{x + 3} = 3x^2 + 11x - 4$

57. $f(x) = 20x^4 + 9x^3 - 14x^2 - 3x$

(a)

$$\begin{array}{r|rrrrr} -1 & 20 & 9 & -14 & -3 & 0 \\ & & -20 & 11 & 3 & 0 \\ \hline & 20 & -11 & -3 & 0 & 0 \end{array}$$

Yes, $x = -1$ is a zero of f.

(b)

$$\begin{array}{r|rrrrr} \frac{3}{4} & 20 & 9 & -14 & -3 & 0 \\ & & 15 & 18 & 3 & 0 \\ \hline & 20 & 24 & 4 & 0 & 0 \end{array}$$

Yes, $x = \frac{3}{4}$ is a zero of f.

(c)

$$\begin{array}{r|rrrrr} 0 & 20 & 9 & -14 & -3 & 0 \\ & & 0 & 0 & 0 & 0 \\ \hline & 20 & 9 & -14 & -3 & 0 \end{array}$$

Yes, $x = 0$ is a zero of f.

(d)

$$\begin{array}{r|rrrrr} 1 & 20 & 9 & -14 & -3 & 0 \\ & & 20 & 29 & 15 & 12 \\ \hline & 20 & 29 & 15 & 12 & 12 \end{array}$$

No, $x = 1$ is not a zero of f.

58. $f(x) = 3x^3 - 8x^2 - 20x + 16$

(a) $$\begin{array}{r|rrrr} 4 & 3 & -8 & -20 & 16 \\ & & 12 & 16 & -16 \\ \hline & 3 & 4 & -4 & 0 \end{array}$$

Yes, $x = 4$ is a zero of f.

(b) $$\begin{array}{r|rrrr} -4 & 3 & -8 & -20 & 16 \\ & & -12 & 80 & -240 \\ \hline & 3 & -20 & 60 & -224 \end{array}$$

No, $x = -4$ is not a zero of f.

(c) $$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -8 & -20 & 16 \\ & & 2 & -4 & -16 \\ \hline & 3 & -6 & -24 & 0 \end{array}$$

Yes, $x = \frac{2}{3}$ is a zero of f.

(d) $$\begin{array}{r|rrrr} -1 & 3 & -8 & -20 & 16 \\ & & -3 & 11 & 9 \\ \hline & 3 & -11 & -9 & 25 \end{array}$$

No, $x = -1$ is not a zero of f.

59. $f(x) = x^4 + 10x^3 - 24x^2 + 20x + 44$

(a) $$\begin{array}{r|rrrrr} -3 & 1 & 10 & -24 & 20 & 44 \\ & & -3 & -21 & 135 & -465 \\ \hline & 1 & 7 & -45 & 155 & -421 \end{array}$$

Thus, $f(-3) = -421$.

(b) $$\begin{array}{r|rrrrr} -1 & 1 & 10 & -24 & 20 & 44 \\ & & -1 & -9 & 33 & -53 \\ \hline & 1 & 9 & -33 & 53 & -9 \end{array}$$

$f(-1) = -9$

60. $g(t) = 2t^5 - 5t^4 - 8t + 20$

(a) $$\begin{array}{r|rrrrrr} -4 & 2 & -5 & 0 & 0 & -8 & 20 \\ & & -8 & 52 & -208 & 832 & -3296 \\ \hline & 2 & -13 & 52 & -208 & 824 & -3276 \end{array}$$

Thus, $g(-4) = -3276$.

(b) $$\begin{array}{r|rrrrrr} \sqrt{2} & 2 & -5 & 0 & 0 & -8 & 20 \\ & & 2\sqrt{2} & -5\sqrt{2} + 4 & -10 + 4\sqrt{2} & -10\sqrt{2} + 8 & -20 \\ \hline & 2 & -5 + 2\sqrt{2} & -5\sqrt{2} + 4 & -10 + 4\sqrt{2} & -10\sqrt{2} & 0 \end{array}$$

Thus, $g(\sqrt{2}) = 0$.

61. $f(x) = x^3 + 4x^2 - 25x - 28$; Factor: $(x - 4)$

(a) $$\begin{array}{r|rrrr} 4 & 1 & 4 & -25 & -28 \\ & & 4 & 32 & 28 \\ \hline & 1 & 8 & 7 & 0 \end{array}$$

Yes, $x - 4$ is a factor of $f(x)$.

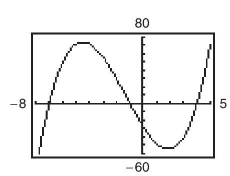
(b) $x^2 + 8x + 7 = (x + 7)(x + 1)$

The remaining factors of f are $(x + 7)$ and $(x + 1)$.

(c) $f(x) = x^3 + 4x^2 - 25x - 28$

$= (x + 7)(x + 1)(x - 4)$

(d) Zeros: $-7, -1, 4$

(e) 

62. $f(x) = 2x^3 + 11x^2 - 21x - 90$

(a) $$\begin{array}{r|rrrr} -6 & 2 & 11 & -21 & -90 \\ & & -12 & 6 & 90 \\ \hline & 2 & -1 & -15 & 0 \end{array}$$

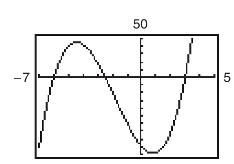
Yes, $(x + 6)$ is a factor of $f(x)$.

(b) $2x^2 - x - 15 = (2x + 5)(x - 3)$

The remaining factors are $(2x + 5)$ and $(x - 3)$.

(c) $f(x) = (2x + 5)(x - 3)(x + 6)$

(d) Zeros: $x = -\frac{5}{2}, 3, -6$

(e) 

63. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

Factors: $(x + 2), (x - 3)$

(a)
$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline & 1 & -6 & 5 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 5 & 12 \\ & & 3 & -9 & -12 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

Both are factors since the remainders are zero.

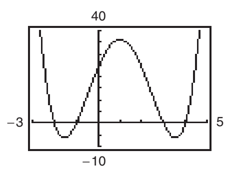
(b) $x^2 - 3x - 4 = (x + 1)(x - 4)$

The remaining factors are $(x + 1)$ and $(x - 4)$.

(c) $f(x) = (x + 1)(x - 4)(x + 2)(x - 3)$

(d) Zeros: $-2, -1, 3, 4$

(e)



64. $f(x) = x^4 - 11x^3 + 41x^2 - 61x + 30$

(a)
$$\begin{array}{r|rrrrr} 2 & 1 & -11 & 41 & -61 & 30 \\ & & 2 & -18 & 46 & -30 \\ \hline & 1 & -9 & 23 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -9 & 23 & -15 \\ & & 5 & -20 & 15 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

Yes, $(x - 2)$ and $(x - 5)$ are both factors of $f(x)$.

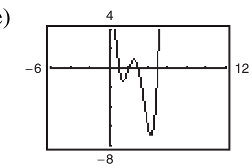
(b) $x^2 - 4x + 3 = (x - 1)(x - 3)$

The remaining factors are $(x - 1)$ and $(x - 3)$.

(c) $f(x) = (x - 1)(x - 3)(x - 2)(x - 5)$

(d) Zeros: $x = 1, 2, 3, 5$

(e)



65. $6 + \sqrt{-4} = 6 + 2i$

66. $3 - \sqrt{-25} = 3 - 5i$

67. $i^2 + 3i = -1 + 3i$

68. $-5i + i^2 = -1 - 5i$

69. $(7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i) = 3 + 7i$

70. $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = -2\left(\frac{\sqrt{2}}{2}i\right) = -\sqrt{2}i$

71. $5i(13 - 8i) = 65i - 40i^2 = 40 + 65i$

72. $$\begin{aligned}(1 + 6i)(5 - 2i) &= 5 - 2i + 30i - 12i^2 \\ &= 5 + 28i + 12 \\ &= 17 + 28i\end{aligned}$$

73. $$\begin{aligned}(10 - 8i)(2 - 3i) &= 20 - 30i - 16i + 24i^2 \\ &= -4 - 46i\end{aligned}$$

74. $$\begin{aligned}i(6 + i)(3 - 2i) &= i(18 - 12i + 3i - 2i^2) \\ &= i(20 - 9i) \\ &= 20i - 9i^2 \\ &= 9 + 20i\end{aligned}$$

75. $$\begin{aligned}\frac{6 + i}{4 - i} &= \frac{6 + i}{4 - i} \cdot \frac{4 + i}{4 + i} \\ &= \frac{24 + 10i + i^2}{16 + 1} \\ &= \frac{23 + 10i}{17} \\ &= \frac{23}{17} + \frac{10}{17}i\end{aligned}$$

76. $$\begin{aligned}\frac{3 + 2i}{5 + i} &= \frac{3 + 2i}{5 + i} \cdot \frac{5 - i}{5 - i} \\ &= \frac{15 - 3i + 10i - 2i^2}{25 - i^2} \\ &= \frac{17 + 7i}{26} \\ &= \frac{17}{26} + \frac{7i}{26}\end{aligned}$$

77. $\dfrac{4}{2-3i}+\dfrac{2}{1+i}=\dfrac{4}{2-3i}\cdot\dfrac{2+3i}{2+3i}+\dfrac{2}{1+i}\cdot\dfrac{1-i}{1-i}$

$$=\frac{8+12i}{4+9}+\frac{2-2i}{1+1}$$

$$=\frac{8}{13}+\frac{12}{13}i+1-i$$

$$=\left(\frac{8}{13}+1\right)+\left(\frac{12}{13}i-i\right)$$

$$=\frac{21}{13}-\frac{1}{13}i$$

78. $\dfrac{1}{2+i}-\dfrac{5}{1+4i}=\dfrac{(1+4i)-5(2+i)}{(2+i)(1+4i)}$

$$=\frac{1+4i-10-5i}{2+8i+i+4i^2}$$

$$=\frac{-9-i}{-2+9i}\cdot\frac{(-2-9i)}{(-2-9i)}$$

$$=\frac{18+81i+2i+9i^2}{4-81i^2}$$

$$=\frac{9+83i}{85}=\frac{9}{85}+\frac{83i}{85}$$

79. $3x^2+1=0$

$$3x^2=-1$$

$$x^2=-\frac{1}{3}$$

$$x=\pm\sqrt{-\frac{1}{3}}$$

$$=\pm\sqrt{\frac{1}{3}}\,i=\pm\frac{\sqrt{3}}{3}i$$

80. $2+8x^2=0$

$$8x^2=-2$$

$$x^2=-\frac{1}{4}$$

$$x=\pm\frac{1}{2}i$$

81. $x^2-2x+10=0$

$$x^2-2x+1=-10+1$$

$$(x-1)^2=-9$$

$$x-1=\pm\sqrt{-9}$$

$$x=1\pm 3i$$

82. $6x^2+3x+27=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-3\pm\sqrt{3^2-4(6)(27)}}{2(6)}$$

$$=\frac{-3\pm\sqrt{-639}}{12}$$

$$=\frac{-3\pm 3i\sqrt{71}}{12}=-\frac{1}{4}\pm\frac{\sqrt{71}}{4}i$$

83. $f(x)=3x(x-2)^2$

Zeros: $x=0, x=2$

84. $f(x)=(x-4)(x+9)^2$

Zeros: $x=-9, 4$

85. $f(x)=x^2-9x+8$

$$=(x-1)(x-8)$$

Zeros: $x=1, x=8$

86. $f(x)=x^3+6x$

$$=x(x^2+6)$$

Zeros: $x=0, \pm\sqrt{6}i$

87. $f(x)=(x+4)(x-6)(x-2i)(x+2i)$

Zeros: $x=-4, x=6, x=2i, x=-2i$

88. $f(x)=(x-8)(x-5)^2(x-3+i)(x-3-i)$

Zeros: $x=5, 8, 3\pm i$

89. $f(x)=-4x^3+8x^2-3x+15$

Possible rational zeros: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}$

90. $f(x)=3x^4+4x^3-5x^2-8$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

91. $f(x) = x^3 - 2x^2 - 21x - 18$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -21 & -18 \\ & & -1 & 3 & 18 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

$$\begin{aligned} x^3 - 2x^2 - 21x - 18 &= (x + 1)(x^2 - 3x - 18) \\ &= (x + 1)(x - 6)(x + 3) \end{aligned}$$

The zeros of $f(x)$ are $x = -1$, $x = 6$, and $x = -3$.

92. $f(x) = 3x^3 - 20x^2 + 7x + 30$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

$$\begin{array}{r|rrrr} -1 & 3 & -20 & 7 & 30 \\ & & -3 & 23 & -30 \\ \hline & 3 & -23 & 30 & 0 \end{array}$$

So, $f(x) = 3x^3 - 20x^2 + 7x + 30$

$$\begin{aligned} &= (x + 1)(3x^2 - 23x + 30) \\ &= (x + 1)(3x - 5)(x - 6) \\ 0 &= (x + 1)(3x - 5)(x - 6). \end{aligned}$$

Zeros: $x = -1, \frac{5}{3}, 6$

93. $f(x) = x^3 - 10x^2 + 17x - 8$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8$

$$\begin{array}{r|rrrr} 1 & 1 & -10 & 17 & -8 \\ & & 1 & -9 & 8 \\ \hline & 1 & -9 & 8 & 0 \end{array}$$

$$\begin{aligned} x^3 - 10x^2 + 17x - 8 &= (x - 1)(x^2 - 9x + 8) \\ &= (x - 1)(x - 1)(x - 8) \\ &= (x - 1)^2(x - 8) \end{aligned}$$

The zeros of $f(x)$ are $x = 1$ and $x = 8$.

94. $f(x) = x^3 + 9x^2 + 24x + 20$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

$$\begin{array}{r|rrrr} -5 & 1 & 9 & 24 & 20 \\ & & -5 & -20 & -20 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

So, $f(x) = x^3 + 9x^2 + 24x + 20$

$$\begin{aligned} &= (x + 5)(x^2 + 4x + 4) \\ &= (x + 5)(x + 2)^2. \end{aligned}$$

Zeros: $x = -5, -2$

95. $f(x) = x^4 + x^3 - 11x^2 + x - 12$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -11 & 1 & -12 \\ & & 3 & 12 & 3 & 12 \\ \hline & 1 & 4 & 1 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 4 & 1 & 4 \\ & & -4 & 0 & -4 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$x^4 + x^3 - 11x^2 + x - 12 = (x - 3)(x + 4)(x^2 + 1)$$

The real zeros of $f(x)$ are $x = 3$, and $x = -4$.

96. $f(x) = 25x^4 + 25x^3 - 154x^2 - 4x + 24$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{8}{5}, \pm\frac{12}{5}, \pm\frac{24}{5}, \pm\frac{1}{25}, \pm\frac{2}{25}, \pm\frac{3}{25}, \pm\frac{4}{25}, \pm\frac{6}{25}, \pm\frac{8}{25}, \pm\frac{12}{25}, \pm\frac{24}{25}$

$$\begin{array}{r|rrrrr} -3 & 25 & 25 & -154 & -4 & 24 \\ & & -75 & 150 & 12 & -24 \\ \hline & 25 & -50 & -4 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 25 & -50 & -4 & 8 \\ & & 50 & 0 & -8 \\ \hline & 25 & 0 & -4 & 0 \end{array}$$

So, $f(x) = 25x^4 + 25x^3 - 154x^2 - 4x + 24$

$$\begin{aligned} &= (x + 3)(x - 2)(25x^2 - 4) \\ &= (x + 3)(x - 2)(5x + 2)(5x - 2). \end{aligned}$$

Zeros: $x = -3, 2, \pm\frac{2}{5}$

97. $f(x) = 3\left(x - \frac{2}{3}\right)(x - 4)\left(x - \sqrt{3}i\right)\left(x + \sqrt{3}i\right)$ Since $\sqrt{3}i$ is a zero, so is $-\sqrt{3}i$.

$= (3x - 2)(x - 4)(x^2 + 3)$ Multiply by 3 to clear the fraction.

$= (3x^2 - 14x + 8)(x^2 + 3)$

$= 3x^4 - 14x^3 + 17x^2 - 42x + 24$

Note: $f(x) = a(3x^4 - 14x^3 + 17x^2 - 42x + 24)$, where a is any real nonzero number, has zeros $\frac{2}{3}$, 4, and $\pm\sqrt{3}i$.

98. Since $1 - 2i$ is a zero and the coefficients are real, $1 + 2i$ must also be a zero.

$f(x) = (x - 2)(x + 3)(x - 1 + 2i)(x - 1 - 2i)$

$= (x^2 + x - 6)[(x - 1)^2 + 4]$

$= (x^2 + x - 6)(x^2 - 2x + 5)$

$= x^4 - x^3 - 3x^2 + 17x - 30$

99. $f(x) = x^3 - 4x^2 + x - 4$, Zero: i

Since i is a zero, so is $-i$.

$$\begin{array}{r|rrrr} i & 1 & -4 & 1 & -4 \\ & & i & -1 - 4i & 4 \\ \hline & 1 & -4 + i & -4i & 0 \end{array}$$

$$\begin{array}{r|rrr} -i & 1 & -4 + i & -4i \\ & & -i & 4i \\ \hline & 1 & -4 & 0 \end{array}$$

$f(x) = (x - i)(x + i)(x - 4)$, Zeros: $x = \pm i, 4$

100. $h(x) = -x^3 + 2x^2 - 16x + 32$

Since $-4i$ is a zero, so is $4i$.

$$\begin{array}{r|rrrr} -4i & -1 & 2 & -16 & 32 \\ & & 4i & 16 - 8i & -32 \\ \hline & -1 & 2 + 4i & -8i & 0 \end{array}$$

$$\begin{array}{r|rrr} 4i & -1 & 2 + 4i & -8i \\ & & -4i & 8i \\ \hline & -1 & 2 & 0 \end{array}$$

$h(x) = (x + 4i)(x - 4i)(-x + 2)$

Zeros: $x = \pm 4i, 2$

101. $g(x) = 2x^4 - 3x^3 - 13x^2 + 37x - 15$, Zero: $2 + i$

Since $2 + i$ is a zero, so is $2 - i$

$$\begin{array}{r|rrrrr} 2 + i & 2 & -3 & -13 & 37 & -15 \\ & & 4 + 2i & 5i & -31 - 3i & 15 \\ \hline & 2 & 1 + 2i & -13 + 5i & 6 - 3i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 - i & 2 & 1 + 2i & -13 + 5i & 6 - 3i \\ & & 4 - 2i & 10 - 5i & -6 + 3i \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$g(x) = [x - (2 + i)][x - (2 - i)](2x^2 + 5x - 3)$

$= (x - 2 - i)(x - 2 + i)(2x - 1)(x + 3)$

Zeros: $x = 2 \pm i, \frac{1}{2}, -3$

102. $f(x) = 4x^4 - 11x^3 + 14x^2 - 6x$

$= x(4x^3 - 11x^2 + 14x - 6)$

One zero is $x = 0$. Since $1 - i$ is a zero, so is $1 + i$.

$$\begin{array}{r|rrrr} 1 - i & 4 & -11 & 14 & -6 \\ & & 4 - 4i & -11 + 3i & 6 \\ \hline & 4 & -7 - 4i & 3 + 3i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 + i & 4 & -7 - 4i & 3 + 3i \\ & & 4 + 4i & -3 - 3i \\ \hline & 4 & -3 & 0 \end{array}$$

$f(x) = x[x - (1 - i)][x - (1 + i)](4x - 3)$

$= x(x - 1 + i)(x - 1 - i)(4x - 3)$

Zeros: $0, \frac{3}{4}, 1 + i, 1 - i$

103. $f(x) = x^3 + 4x^2 - 5x$

$= x(x^2 + 4x - 5)$

$= x(x + 5)(x - 1)$

Zeros: $x = 0, -5, 1$

104. $g(x) = x^3 - 7x^2 + 36$

$$\begin{array}{r|rrrr} -2 & 1 & -7 & 0 & 36 \\ & & -2 & 18 & -36 \\ \hline & 1 & -9 & 18 & 0 \end{array}$$

The zeros of $x^2 - 9x + 18 = (x - 3)(x - 6)$ are $x = 3, 6$. The zeros of $g(x)$ are $-2, 3, 6$.

$g(x) = (x + 2)(x - 3)(x - 6)$

105. $g(x) = x^4 + 4x^3 - 3x^2 + 40x + 208$, Zero: $x = -4$

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & -3 & 40 & 208 \\ & & -4 & 0 & 12 & -208 \\ \hline & 1 & 0 & -3 & 52 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -3 & 52 \\ & & -4 & 16 & -52 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

$g(x) = (x + 4)^2(x^2 - 4x + 13)$

By the Quadratic Formula the zeros of $x^2 - 4x + 13$ are $x = 2 \pm 3i$. The zeros of $g(x)$ are $x = -4$ of multiplicity 2, and $x = 2 \pm 3i$.

$$\begin{aligned} g(x) &= (x + 4)^2[x - (2 + 3i)][x - (2 - 3i)] \\ &= (x + 4)^2(x - 2 - 3i)(x - 2 + 3i) \end{aligned}$$

106. $f(x) = x^4 + 8x^3 + 8x^2 - 72x - 153$

$$\begin{array}{r|rrrrr} 3 & 1 & 8 & 8 & -72 & -153 \\ & & 3 & 33 & 123 & 153 \\ \hline & 1 & 11 & 41 & 51 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 11 & 41 & 51 \\ & & -3 & -24 & -51 \\ \hline & 1 & 8 & 17 & \end{array}$$

By the Quadratic Formula, the zeros of $x^2 + 8x + 17$ are

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(1)(17)}}{2(1)} = \frac{-8 \pm \sqrt{-4}}{2} = -4 \pm i.$$

The zeros of $f(x)$ are $-3, 3, -4 - i, -4 + i$.

$f(x) = (x + 3)(x - 3)(x + 4 - i)(x + 4 + i)$

107. $g(x) = 5x^3 + 3x^2 - 6x + 9$

$g(x)$ has two variations in sign, so g has either two or no positive real zeros.

$g(-x) = -5x^3 + 3x^2 + 6x + 9$

$g(-x)$ has one variation in sign, so g has one negative real zero.

108. $h(x) = -2x^5 + 4x^3 - 2x^2 + 5$

$h(x)$ has three variations in sign, so h has either three or one positive real zeros.

$$\begin{aligned} h(-x) &= -2(-x)^5 + 4(-x)^3 - 2(-x)^2 + 5 \\ &= 2x^5 - 4x^3 - 2x^2 + 5 \end{aligned}$$

$h(-x)$ has two variations in sign, so h has either two or no negative real zeros.

109. $f(x) = 4x^3 - 3x^2 + 4x - 3$

(a)
$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

Since the last row has all positive entries, $x = 1$ is an upper bound.

(b)
$$\begin{array}{r|rrrr} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & & -1 & 1 & -\frac{5}{4} \\ \hline & 4 & -4 & 5 & -\frac{17}{4} \end{array}$$

Since the last row entries alternate in sign, $x = -\frac{1}{4}$ is a lower bound.

110. $g(x) = 2x^3 - 5x^2 - 14x + 8$

(a)
$$\begin{array}{r|rrrr} 8 & 2 & -5 & -14 & 8 \\ & & 16 & 88 & 592 \\ \hline & 2 & 11 & 74 & 600 \end{array}$$

Since the last row has all positive entries, $x = 8$ is an upper bound.

(b)
$$\begin{array}{r|rrrr} -4 & 2 & -5 & -14 & 8 \\ & & -8 & 52 & -152 \\ \hline & 2 & -13 & 38 & -144 \end{array}$$

Since the last row entries alternate in sign, $x = 4$ is a lower bound.

111. $f(x) = \dfrac{5x}{x + 12}$

Domain: all real numbers x except $x = -12$

112. $f(x) = \dfrac{3x^2}{1 + 3x}$

$1 + 3x = 0$

$3x = -1$

$x = -\dfrac{1}{3}$

Domain: all real numbers x except $x = -\frac{1}{3}$

113. $f(x) = \dfrac{8}{x^2 - 10x + 24}$

$= \dfrac{8}{(x - 4)(x - 6)}$

Domain: all real numbers x except $x = 4$ and $x = 6$

114. $f(x) = \dfrac{x^2 - x - 2}{x^2 + 4}$

Domain: all real numbers

115. $f(x) = \dfrac{4}{x + 3}$

Vertical asymptote: $x = -3$

Horizontal asymptote: $y = 0$

116. $f(x) = \dfrac{2x^2 + 5x - 3}{x^2 + 2}$

Vertical asymptote: none

Horizontal asymptote: $y = 2$

117. $h(x) = \dfrac{2x - 10}{x^2 - 2x - 15}$

$= \dfrac{2(x - 5)}{(x + 3)(x - 5)}$

$= \dfrac{2}{x + 3}, \ x \neq 5$

Vertical asymptote: $x = -3$

Horizontal asymptote: $y = 0$

118. $h(x) = \dfrac{x^3 - 4x^2}{x^2 + 3x + 2} = \dfrac{x^2(x - 4)}{(x + 2)(x + 1)}$

Vertical asymptotes: $x = -2, x = -1$

Horizontal asymptotes: none

119. $f(x) = \dfrac{-5}{x^2}$

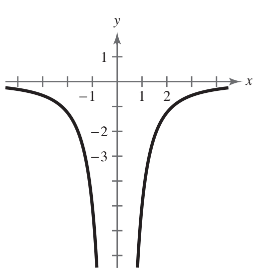
(a) Domain: all real numbers x except $x = 0$

(b) No intercepts

(c) Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$

(d)

x	± 3	± 2	± 1
y	$-\frac{5}{9}$	$-\frac{5}{4}$	-5



120. $f(x) = \dfrac{4}{x}$

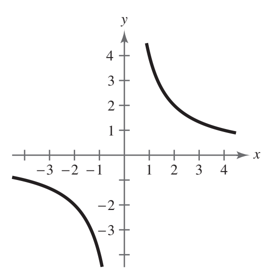
(a) Domain: all real numbers x except $x = 0$

(b) No intercepts

(c) Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$

(d)

x	-3	-2	-1	1	2	3
y	$-\frac{4}{3}$	-2	-4	4	2	$\frac{4}{3}$



121. $g(x) = \dfrac{2 + x}{1 - x} = -\dfrac{x + 2}{x - 1}$

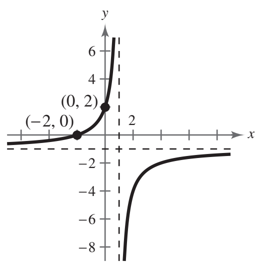
(a) Domain: all real numbers x except $x = 1$

(b) x-intercept: $(-2, 0)$
y-intercept: $(0, 2)$

(c) Vertical asymptote: $x = 1$
Horizontal asymptote: $y = -1$

(d)

x	-1	0	2	3
y	$\frac{1}{2}$	2	-4	$-\frac{5}{2}$



122. $h(x) = \dfrac{x - 3}{x - 2}$

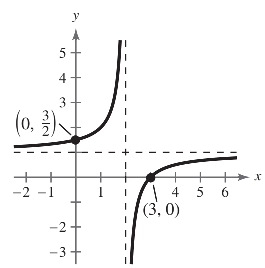
(a) Domain: all real numbers x except $x = 2$

(b) x-intercept: $(3, 0)$
y-intercept: $\left(0, \dfrac{3}{2}\right)$

(c) Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 1$

(d)

x	-1	0	1	3	4	5
y	$\frac{4}{3}$	$\frac{3}{2}$	2	0	$\frac{1}{2}$	$\frac{2}{3}$



123. $p(x) = \dfrac{x^2}{x^2 + 1}$

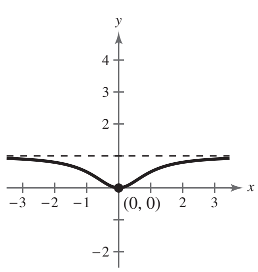
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 1$

(d)

x	± 3	± 2	± 1	0
y	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{1}{2}$	0



124. $f(x) = \dfrac{2x}{x^2 + 4}$

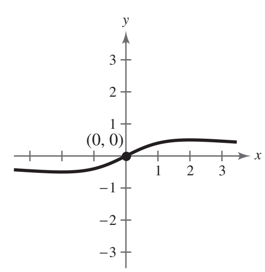
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

x	-2	-1	0	1	2
y	$-\frac{1}{2}$	$-\frac{2}{5}$	0	$\frac{2}{5}$	$\frac{1}{2}$



125. $f(x) = \dfrac{x}{x^2 + 1}$

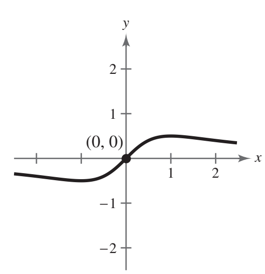
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

x	-2	-1	0	1	2
y	$-\frac{2}{5}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{2}{5}$



126. $h(x) = \dfrac{4}{(x-1)^2}$

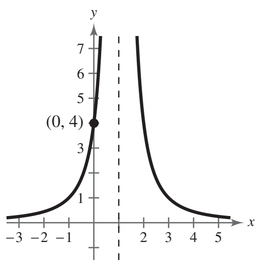
(a) Domain: all real numbers x except $x = 1$

(b) y-intercept: $(0, 4)$

(c) Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 0$

(d)

x	-2	-1	0	2	3	4
y	$\frac{4}{9}$	1	4	4	1	$\frac{4}{9}$



127. $f(x) = \dfrac{-6x^2}{x^2 + 1}$

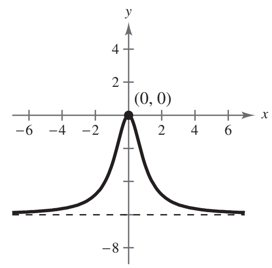
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = -6$

(d)

x	± 3	± 2	± 1	0
y	$-\frac{27}{5}$	$-\frac{24}{5}$	-3	0



128. $y = \dfrac{2x^2}{x^2 - 4}$

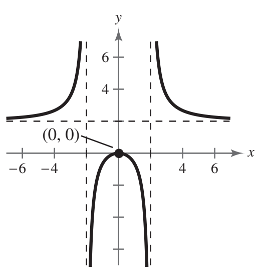
(a) Domain: all real numbers x except $x = \pm 2$

(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = 2, x = -2$
Horizontal asymptote: $y = 2$

(d)

x	± 5	± 4	± 3	± 1	0
y	$\frac{50}{21}$	$\frac{8}{3}$	$\frac{18}{5}$	$-\frac{2}{3}$	0



129. $f(x) = \dfrac{6x^2 - 11x + 3}{3x^2 - x}$

$$= \frac{(3x-1)(2x-3)}{x(3x-1)} = \frac{2x-3}{x}, \quad x \neq \frac{1}{3}$$

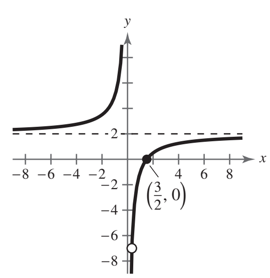
(a) Domain: all real numbers x except $x = 0$ and $x = \dfrac{1}{3}$

(b) x-intercept: $\left(\dfrac{3}{2}, 0\right)$
y-intercept: none

(c) Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 2$

(d)

x	-2	-1	1	2	3	4
y	$\frac{7}{2}$	5	-1	$\frac{1}{2}$	1	$\frac{5}{4}$



130. $f(x) = \dfrac{6x^2 - 7x + 2}{4x^2 - 1}$

$= \dfrac{(2x - 1)(3x - 2)}{(2x - 1)(2x + 1)} = \dfrac{3x - 2}{2x + 1},\ x \neq \dfrac{1}{2}$

(a) Domain: all real numbers x except $x \neq \pm\dfrac{1}{2}$

(b) y-intercept: $(0, -2)$

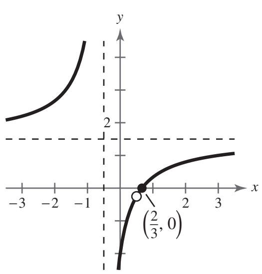
x-intercept: $\left(\dfrac{2}{3}, 0\right)$

(c) Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = \dfrac{3}{2}$

(d)

x	-3	-2	-1	0	$\frac{2}{3}$	1	2
y	$\frac{11}{5}$	$\frac{8}{3}$	5	-2	0	$\frac{1}{3}$	$\frac{4}{5}$



131. $f(x) = \dfrac{2x^3}{x^2 + 1} = 2x - \dfrac{2x}{x^2 + 1}$

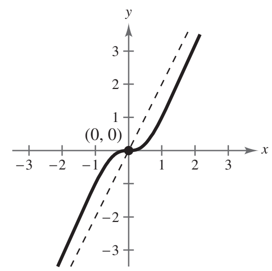
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Slant asymptote: $y = 2x$

(d)

x	-2	-1	0	1	2
y	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$



132. $f(x) = \dfrac{x^2 + 1}{x + 1}$

(a) Domain: all real numbers x except $x = -1$

(b) y-intercept: $(0, 1)$

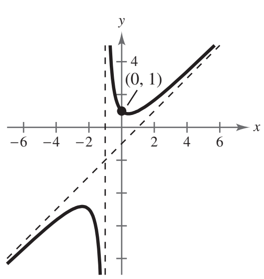
(c) Vertical asymptote: $x = -1$

Using long division, $f(x) = \dfrac{x^2 + 1}{x + 1} = x - 1 + \dfrac{2}{x + 1}$.

Slant asymptote: $y = x - 1$

(d)

x	-6	-2	$-\frac{3}{2}$	$-\frac{1}{2}$	0	4
y	$-\frac{37}{5}$	-5	$-\frac{13}{2}$	$\frac{5}{2}$	1	$\frac{17}{5}$



133. $f(x) = \dfrac{3x^3 - 2x^2 - 3x + 2}{3x^2 - x - 4}$

$= \dfrac{(3x - 2)(x + 1)(x - 1)}{(3x - 4)(x + 1)}$

$= \dfrac{(3x - 2)(x - 1)}{3x - 4}$

$= x - \dfrac{1}{3} + \dfrac{2/3}{3x - 4}, \quad x \neq -1$

(a) Domain: all real numbers x except $x = -1, x = \dfrac{4}{3}$

(b) x-intercepts: $(1, 0)$ and $\left(\dfrac{2}{3}, 0\right)$

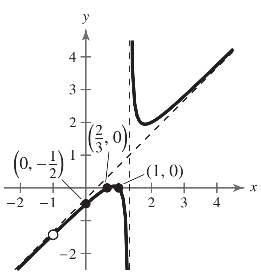
y-intercept: $\left(0, -\dfrac{1}{2}\right)$

(c) Vertical asymptote: $x = \dfrac{4}{3}$

Slant asymptote: $y = x - \dfrac{1}{3}$

(d)

x	-3	-2	0	1	2	3
y	$-\frac{44}{13}$	$-\frac{12}{5}$	$-\frac{1}{2}$	0	2	$\frac{14}{5}$



134. $f(x) = \dfrac{3x^3 - 4x^2 - 12x + 16}{3x^2 + 5x - 2}$

$= \dfrac{(x - 2)(x + 2)(3x - 4)}{(x + 2)(3x - 1)}$

$= \dfrac{(x - 2)(3x - 4)}{3x - 1}, x \neq -2$

(a) Domain: all real x except $x = -2$ or $x = \dfrac{1}{3}$

(b) y-intercept: $(0, -8)$

x-intercepts: $\left(\dfrac{4}{3}, 0\right), (2, 0)$

(c) Vertical asymptote: $x = \dfrac{1}{3}$

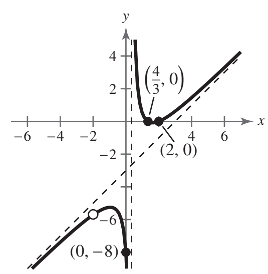
Using long division,

$f(x) = \dfrac{3x^2 - 10x + 8}{3x - 1} = x - 3 + \dfrac{5}{3x - 1}.$

Slant asymptote: $y = x - 3$

(d)

x	-4	-1	0	1	2	4
y	$-\frac{96}{13}$	$-\frac{21}{4}$	-8	$\frac{1}{2}$	0	$\frac{16}{11}$



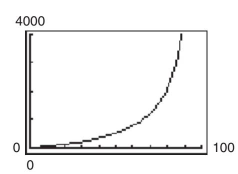
135. $\overline{C} = \dfrac{C}{x} = \dfrac{0.5x + 500}{x}, \ 0 < x$

Horizontal asymptote: $\overline{C} = \dfrac{0.5}{1} = 0.5$

As x increases, the average cost per unit approaches the horizontal asymptote, $\overline{C} = 0.5 = \$0.50$.

136. $C = \dfrac{528p}{100 - p}, \ 0 \leq p < 100$

(a)



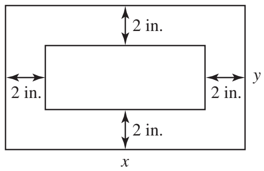
(b) When $p = 25$, $C = \dfrac{528(25)}{100 - 25} = \$176$ million.

When $p = 50$, $C = \dfrac{528(50)}{100 - 50} = \$528$ million.

When $p = 75$, $C = \dfrac{528(75)}{100 - 75} = \$1584$ million.

(c) As $p \longrightarrow 100$, $C \longrightarrow \infty$. No, it is not possible.

137. (a)



(b) The area of print is $(x-4)(y-4)$, which is 30 square inches.

$$(x-4)(y-4) = 30$$

$$y - 4 = \frac{30}{x-4}$$

$$y = \frac{30}{x-4} + 4$$

$$y = \frac{30 + 4(x-4)}{x-4}$$

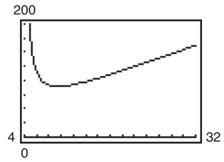
$$y = \frac{4x+14}{x-4}$$

$$y = \frac{2(2x+7)}{x-4}$$

$$\text{Total area} = xy = x\left[\frac{2(2x+7)}{x-4}\right] = \frac{2x(2x+7)}{x-4}$$

(c) Because the horizontal margins total 4 inches, x must be greater than 4 inches. The domain is $x > 4$.

(d)



The minimum area occurs when $x \approx 9.477$ inches, so

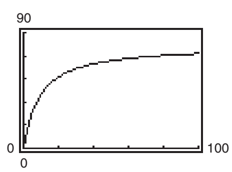
$$y \approx \frac{2(2 \cdot 9.477 + 7)}{9.477 - 4} \approx 9.477 \text{ inches.}$$

The least amount of paper used is for a page size of about 9.48 inches by 9.48 inches.

138. $y = \dfrac{18.47x - 2.96}{0.23x + 1},\ 0 < x$

The limiting amount of CO_2 uptake is determined by the horizontal asymptote,

$$y = \frac{18.47}{0.23} \approx 80.3 \text{ mg/dm}^2\text{/hr.}$$



139. $6x^2 + 5x < 4$

$$6x^2 + 5x - 4 < 0$$

$$(3x+4)(2x-1) < 0$$

Critical numbers: $x = -\frac{4}{3}, x = \frac{1}{2}$

Test intervals: $\left(-\infty, -\frac{4}{3}\right), \left(-\frac{4}{3}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

Test: Is $(3x+4)(2x-1) < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $\left(-\frac{4}{3}, \frac{1}{2}\right)$

140. $2x^2 + x \ge 15$

$$2x^2 + x - 15 \ge 0$$

$$(2x-5)(x+3) \ge 0$$

Critical numbers: $x = \frac{5}{2}, x = -3$

Test intervals: $(-\infty, -3) \implies (2x-5)(x+3) > 0$

$\left(-3, \frac{5}{2}\right) \implies (2x-5)(x+3) < 0$

$\left(\frac{5}{2}, \infty\right) \implies (2x-5)(x+3) > 0$

Solution interval: $(-\infty, -3] \cup \left[\frac{5}{2}, \infty\right)$

141. $x^3 - 16x \ge 0$

$$x(x+4)(x-4) \ge 0$$

Critical numbers: $x = 0, x = \pm 4$

Test intervals: $(-\infty, -4), (-4, 0), (0, 4), (4, \infty)$

Test: Is $x(x+4)(x-4) \ge 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $[-4, 0] \cup [4, \infty)$.

142. $12x^3 - 20x^2 < 0$

$4x^2(3x - 5) < 0$

Critical numbers: $x = 0, x = \frac{5}{3}$

Test intervals: $(-\infty, 0) \implies 12x^3 - 20x^2 < 0$

$\left(0, \frac{5}{3}\right) \implies 12x^3 - 20x^2 < 0$

$\left(\frac{5}{3}, \infty\right) \implies 12x^3 - 20x^2 > 0$

Solution interval: $(-\infty, 0) \cup \left(0, \frac{5}{3}\right)$

143. $$\frac{2}{x+1} \le \frac{3}{x-1}$$

$$\frac{2(x-1) - 3(x+1)}{(x+1)(x-1)} \le 0$$

$$\frac{2x - 2 - 3x - 3}{(x+1)(x-1)} \le 0$$

$$\frac{-(x+5)}{(x+1)(x-1)} \le 0$$

Critical numbers: $x = -5,\ x = \pm 1$

Test intervals: $(-\infty, -5), (-5, -1), (-1, 1), (1, \infty)$

Test: Is $\dfrac{-(x+5)}{(x+1)(x-1)} \le 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $[-5, -1) \cup (1, \infty)$

144. $\dfrac{x-5}{3-x} < 0$

Critical numbers: $x = 5, x = 3$

Test intervals: $(-\infty, 3) \implies \dfrac{x-5}{3-x} < 0$

$(3, 5) \implies \dfrac{x-5}{3-x} > 0$

$(5, \infty) \implies \dfrac{x-5}{3-x} < 0$

Solution intervals: $(-\infty, 3) \cup (5, \infty)$

145. $$\frac{x^2 + 7x + 12}{x} \ge 0$$

$$\frac{(x+4)(x+3)}{x} \ge 0$$

Critical numbers: $x = -4, x = -3, x = 0$

Test intervals: $(-\infty, -4), (-4, -3), (-3, 0), (0, \infty)$

Test: Is $\dfrac{(x+4)(x+3)}{x} \ge 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is: $[-4, -3] \cup (0, \infty)$

146. $$\frac{1}{x-2} > \frac{1}{x}$$

$$\frac{1}{x-2} - \frac{1}{x} > 0$$

Critical numbers: $x = 2, x = 0$

Test intervals: $(-\infty, 0) \implies \dfrac{1}{x-2} - \dfrac{1}{x} > 0$

$(0, 2) \implies \dfrac{1}{x-2} - \dfrac{1}{x} < 0$

$(2, \infty) \implies \dfrac{1}{x-2} - \dfrac{1}{x} > 0$

Solution interval: $(-\infty, 0) \cup (2, \infty)$

147. $5000(1 + r)^2 > 5500$

$(1 + r)^2 > 1.1$

$1 + r > 1.0488$

$r > 0.0488$

$r > 4.9\%$

148. $$P = \frac{1000(1 + 3t)}{5 + t}$$

$$2000 \le \frac{1000(1 + 3t)}{5 + t}$$

$$2000(5 + t) \le 1000(1 + 3t)$$

$$10{,}000 + 2000t \le 1000 + 3000t$$

$$-1000t \le -9000$$

$$t \ge 9 \text{ days}$$

149. False. A fourth-degree polynomial can have at most four zeros and complex zeros occur in conjugate pairs.

150. False. (See Exercise 123.) The domain of

$$f(x) = \frac{1}{x^2 + 1}$$

is the set of all real numbers x.

151. The maximum (or minimum) value of a quadratic function is located at its graph's vertex. To find the vertex, either write the equation in standard form or use the formula

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

If the leading coefficient is positive, the vertex is a minimum. If the leading coefficient is negative, the vertex is a maximum.

152. Answers will vary. Sample answer:

Polynomials of degree $n > 0$ with real coefficients can be written as the product of linear and quadratic factors with real coefficients, where the quadratic factors have no real zeros.

Setting the factors equal to zero and solving for the variable can find the zeros of a polynomial function.

To solve an equation is to find all the values of the variable for which the equation is true.

153. An asymptote of a graph is a line to which the graph becomes arbitrarily close as x increases or decreases without bound.

Problem Solving for Chapter 2

1. $f(x) = ax^3 + bx^2 + cx + d$

$$\begin{array}{r|l}
 & ax^2 + (ak + b)x + (ak^2 + bk + c) \\
\hline
x - k & ax^3 + bx^2 \quad + cx \quad + d \\
 & \underline{ax^3 - akx^2} \\
 & (ak + b)x^2 + cx \\
 & \underline{(ak + b)x^2 - (ak^2 + bk)x} \\
 & (ak^2 + bk + c)x + d \\
 & \underline{(ak^2 + bk + c)x - (ak^3 + bk^2 + ck)} \\
 & (ak^3 + bk^2 + ck + d)
\end{array}$$

Thus, $f(x) = ax^3 + bx^2 + cx + d = (x - k)[ax^2 + (ak + b)x + (ak^2 + bx + c)] + ak^3 + bk^2 + ck + d$ and $f(k) = ak^3 + bk^2 + ck + d$. Since the remainder $r = ak^3 + bk^2 + ck + d$, $f(k) = r$.

2. (a)

y	$y^3 + y^2$
1	2
2	12
3	36
4	80
5	150
6	252
7	392
8	576
9	810
10	1100

(b) $x^3 + x^2 = 252 \Rightarrow x = 6$

(c) $x^3 + 2x^2 = 288;\ a = 1, b = 2 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{1}{8}$

$$\frac{1}{8}x^3 + \frac{1}{8}(2x^2) = \frac{1}{8}(288)$$

$$\left(\frac{x}{2}\right)^3 + \left(\frac{x}{2}\right)^2 = 36 \Rightarrow \frac{x}{2} = 3 \Rightarrow x = 6$$

(d) $3x^3 + x^2 = 90;\ a = 3, b = 1 \Rightarrow \dfrac{a^2}{b^3} = 9$

$$9(3x^3) + 9x^2 = 9(90)$$

$$(3x)^3 + (3x)^2 = 810 \Rightarrow 3x = 9 \Rightarrow x = 3$$

(e) $2x^3 + 5x^2 = 2500;\ a = 2, b = 5 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{4}{125}$

$$\frac{4}{125}(2x^3) + \frac{4}{125}(5x^2) = \frac{4}{125}(2500)$$

$$\left(\frac{2x}{5}\right)^3 + \left(\frac{2x}{5}\right)^2 = 80 \Rightarrow \frac{2x}{5} = 4 \Rightarrow x = 10$$

(f) $7x^3 + 6x^2 = 1728;\ a = 7, b = 6 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{49}{216}$

$$\frac{49}{216}(7x^3) + \frac{49}{216}(6x^2) = \frac{49}{216}(1728)$$

$$\left(\frac{7x}{6}\right)^3 + \left(\frac{7x}{6}\right)^2 = 392 \Rightarrow \frac{7x}{6} = 7 \Rightarrow x = 6$$

(g) $10x^3 + 3x^2 = 297;\ a = 10, b = 3 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{100}{27}$

$$\frac{100}{27}(10x^3) + \frac{100}{27}(3x^2) = \frac{100}{27}(297)$$

$$\left(\frac{10x}{3}\right)^3 + \left(\frac{10x}{3}\right)^2 = 1100 \Rightarrow \frac{10x}{3} = 10 \Rightarrow x = 3$$

3. $V = l \cdot w \cdot h = x^2(x + 3)$

$x^2(x + 3) = 20$

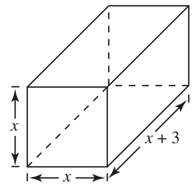
$x^3 + 3x^2 - 20 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & 0 & -20 \\ & & 2 & 10 & 20 \\ \hline & 1 & 5 & 10 & 0 \end{array}$$

$(x - 2)(x^2 + 5x + 10) = 0$

$x = 2 \text{ or } x = \dfrac{-5 \pm \sqrt{15}i}{2}$



Choosing the real positive value for x we have: $x = 2$ and $x + 3 = 5$. The dimensions of the mold are 2 inches $\times$ 2 inches $\times$ 5 inches.

4. False. Since $f(x) = d(x)q(x) + r(x)$, we have $\dfrac{f(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

The statement should be corrected to read $f(-1) = 2$ since $\dfrac{f(x)}{x+1} = q(x) + \dfrac{f(-1)}{x+1}$.

5. (a) $y = ax^2 + bx + c$

$(0, -4)$: $-4 = a(0)^2 + b(0) + c$

$-4 = c$

$(4, 0)$: $0 = a(4)^2 + b(4) - 4$

$0 = 16a + 4b - 4 = 4(4a + b - 1)$

$0 = 4a + b - 1$ or $b = 1 - 4a$

$(1, 0)$: $0 = a(1)^2 + b(1) - 4$

$4 = a + b$

$4 = a + (1 - 4a)$

$4 = 1 - 3a$

$-3 = -3a$

$a = -1$

$b = 1 - 4(-1) = 5$

$y = -x^2 + 5x - 4$

(b) Enter the data points $(0, -4)$, $(1, 0)$, $(2, 2)$, $(4, 0)$, $(6, -10)$ and use the regression feature to obtain $y = -x^2 + 5x - 4$.

6. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$

Slope of tangent line is less than 5.

(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$

Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$

Slope of tangent line is less than 4.1.

(d) Slope $= \dfrac{f(2 + h) - f(2)}{(2 + h) - 2}$

$= \dfrac{(2 + h)^2 - 4}{h}$

$= \dfrac{4h + h^2}{h}$

$= 4 + h, h \neq 0$

(e) Slope $= 4 + h, \quad h \neq 0$

$4 + (-1) = 3$

$4 + 1 = 5$

$4 + 0.1 = 4.1$

The results are the same as in (a)–(c).

(f) Letting h get closer and closer to 0, the slope approaches 4. Hence, the slope at $(2, 4)$ is 4.

7. $f(x) = (x - k)q(x) + r$

(a) Cubic, passes through $(2, 5)$, rises to the right

One possibility:

$$f(x) = (x - 2)x^2 + 5$$
$$= x^3 - 2x^2 + 5$$

(b) Cubic, passes through $(-3, 1)$, falls to the right

One possibility:

$$f(x) = -(x + 3)x^2 + 1$$
$$= -x^3 - 3x^2 + 1$$

8. (a) $z_m = \dfrac{1}{z}$

$$= \frac{1}{1+i} = \frac{1}{1+i} \cdot \frac{1-i}{1-i}$$
$$= \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$$

(b) $z_m = \dfrac{1}{z}$

$$= \frac{1}{3-i} = \frac{1}{3-i} \cdot \frac{3+i}{3+i}$$
$$= \frac{3+i}{10} = \frac{3}{10} + \frac{1}{10}i$$

(c) $z_m = \dfrac{1}{z} = \dfrac{1}{-2+8i}$

$$= \frac{1}{-2+8i} \cdot \frac{-2-8i}{-2-8i}$$
$$= \frac{-2-8i}{68} = -\frac{1}{34} - \frac{2}{17}i$$

9. $(a + bi)(a - bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$

Since a and b are real numbers, $a^2 + b^2$ is also a real number.

10. $f(x) = \dfrac{ax + b}{cx + d}$

Vertical asymptote: $x = -\dfrac{d}{c}$

Horizontal asymptote: $y = \dfrac{a}{c}$

(i) $a > 0, b < 0, c > 0, d < 0$

Both the vertical asymptote and the horizontal asymptote are positive. Matches graph (d).

(ii) $a > 0, b > 0, c < 0, d < 0$

Both the vertical asymptote and the horizontal asymptote are negative. Matches graph (b).

(iii) $a < 0, b > 0, c > 0, d < 0$

The vertical asymptote is positive and the horizontal asymptote is negative. Matches graph (a).

(iv) $a > 0, b < 0, c > 0, d > 0$

The vertical asymptote is negative and the horizontal asymptote is positive. Matches graph (c).

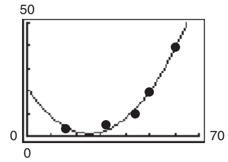
11. $f(x) = \dfrac{ax}{(x - b)^2}$

(a) $b \neq 0 \implies x = b$ is a vertical asymptote. a causes a vertical stretch if $|a| > 1$ and a vertical shrink if $0 < |a| < 1$. For $|a| > 1$, the graph becomes wider as $|a|$ increases. When a is negative the graph is reflected about the x-axis.

(b) $a \neq 0$. Varying the value of b varies the vertical asymptote of the graph of f. For $b > 0$, the graph is translated to the right. For $b < 0$, the graph is reflected in the x-axis and is translated to the left.

12. (a)

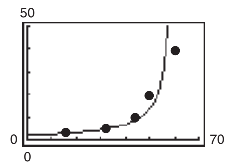
Age, x	Near point, y
16	3.0
32	4.7
44	9.8
50	19.7
60	39.4



$y \approx 0.0313x^2 - 1.586x + 21.02$

(b) $\dfrac{1}{y} \approx -0.007x + 0.44$

$$y \approx \frac{1}{-0.007x + 0.44}$$



(c)

Age, x	Near point, y	Quadratic Model	Rational Model
16	3.0	3.66	3.05
32	4.7	2.32	4.63
44	9.8	11.83	7.58
50	19.7	19.97	11.11
60	39.4	38.54	50.00

The models are fairly good fits to the data. The quadratic model seems to be a better fit for older ages and the rational model a better fit for younger ages.

(d) For $x = 25$, the quadratic model yields $y \approx 0.9325$ inches and the rational model yields $y \approx 3.774$ inches.

(e) The reciprocal model cannot be used to predict the near point for a person who is 70 years old because it results in a negative value $(y \approx -20)$. The quadratic model yields $y \approx 63.37$ inches.

Chapter 2 Practice Test

1. Sketch the graph of $f(x) = x^2 - 6x + 5$ and identify the vertex and the intercepts.

2. Find the number of units x that produce a minimum cost C if
$C = 0.01x^2 - 90x + 15{,}000$.

3. Find the quadratic function that has a maximum at $(1, 7)$ and passes through the point $(2, 5)$.

4. Find two quadratic functions that have x-intercepts $(2, 0)$ and $\left(\frac{4}{3}, 0\right)$.

5. Use the leading coefficient test to determine the right and left end behavior of the graph of the polynomial function $f(x) = -3x^5 + 2x^3 - 17$.

6. Find all the real zeros of $f(x) = x^5 - 5x^3 + 4x$.

7. Find a polynomial function with 0, 3, and -2 as zeros.

8. Sketch $f(x) = x^3 - 12x$.

9. Divide $3x^4 - 7x^2 + 2x - 10$ by $x - 3$ using long division.

10. Divide $x^3 - 11$ by $x^2 + 2x - 1$.

11. Use synthetic division to divide $3x^5 + 13x^4 + 12x - 1$ by $x + 5$.

12. Use synthetic division to find $f(-6)$ given $f(x) = 7x^3 + 40x^2 - 12x + 15$.

13. Find the real zeros of $f(x) = x^3 - 19x - 30$.

14. Find the real zeros of $f(x) = x^4 + x^3 - 8x^2 - 9x - 9$.

15. List all possible rational zeros of the function $f(x) = 6x^3 - 5x^2 + 4x - 15$.

16. Find the rational zeros of the polynomial $f(x) = x^3 - \frac{20}{3}x^2 + 9x - \frac{10}{3}$.

17. Write $f(x) = x^4 + x^3 + 5x - 10$ as a product of linear factors.

18. Find a polynomial with real coefficients that has 2, $3 + i$, and $3 - 2i$ as zeros.

19. Use synthetic division to show that $3i$ is a zero of $f(x) = x^3 + 4x^2 + 9x + 36$.

20. Sketch the graph of $f(x) = \dfrac{x - 1}{2x}$ and label all intercepts and asymptotes.

21. Find all the asymptotes of $f(x) = \dfrac{8x^2 - 9}{x^2 + 1}$.

22. Find all the asymptotes of $f(x) = \dfrac{4x^2 - 2x + 7}{x - 1}$.

23. Given $z_1 = 4 - 3i$ and $z_2 = -2 + i$, find the following:

(a) $z_1 - z_2$

(b) $z_1 z_2$

(c) z_1/z_2

24. Solve the inequality: $x^2 - 49 \le 0$

25. Solve the inequality: $\dfrac{x + 3}{x - 7} \ge 0$